MULTICARRIER DIVERSITY IN RANDOM ACCESS NETWORKS

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SUMMARY

Random access schemes are primarily used for data transmission in the uplink of cellular networks. Every user in a random access network is programmed to follow a predetermined transmit control policy that is designed to achieve optimal network performance. This approach, however, is not very efficient in cellular networks where channel conditions vary from time to time. Employing a fixed transmission policy may not guarantee optimal performance. To alleviate this problem, recently, channel aware random access schemes have been proposed wherein information available at the *physical* (PHY) layer is utilized at the higher layers to maximize the network throughput. Such a cross-layer approach naturally has its share of challenges and problems.

The objective of the proposed research is to study the effect of multicarrier diversity on channel aware random access schemes. First, we describe two generic random access schemes - *channel aware multicarrier random access* (CAMCRA) and *no selection random access* (NS-RA) for multicarrier networks. The former is based on judicious carrier selection and exploits multicarrier diversity while the latter does not perform carrier selection. For illustration purposes, we consider the well-known Aloha protocol and study the effect of channel state imperfection on the overall network throughput. We show that networks employing the NS-RA scheme are extremely sensitive to channel measurement errors. This is in contrast to the performance of the CAMCRA scheme that maintains the same order of throughput in the presence of channel measurement errors. More precisely, the average asymptotic throughput of the NS-RA scheme under uncertain channel conditions is *zero*. The CAMCRA scheme, however, is very robust to estimation errors and maintains the same order of throughput.

Next, we investigate the stability region of two user Aloha network following the CAMCRA scheme. We derive upper and lower bounds on the stability region and compare with the classical result of Tysabakov and Mikhailov for the NS-RA scheme. Specifically, we

see how the stability region is enlarged by exploiting multicarrier diversity and performing carrier selection based on instantaneous channel gains. So far, we have considered collision channel models where simultaneous transmission of more than one packet results in no successful transmission. In practical fading channels, due to the varying level of the received powers, it is possible sometimes to receive at least one packet even in the presence of multiple interferers. We then study the effect of the CAMCRA scheme on such capture networks.

In conclusion, in our research, we have exploited the presence of multiple carriers at the PHY layer to improve the robustness of random access networks. To that end, we have considered two popular models – collision and capture – and shown improved stability properties by judicious carrier selection. Apart from that we have also considered throughput as a performance metric to compare the effectiveness of proposed schemes. In general, we observe that networks employing carrier selection result in higher throughput than those that do not. This is because of the multicarrier diversity benefits that results from carrier selection based on instantaneous channel gains.

CHAPTER I

INTRODUCTION

In the layered network architecture, currently, each layer performs operations independent of the other layers to keep the design simple. With increasingly diverse QoS requirements this method becomes more and more inefficient and new ways to improve the network performance are sought. This has resulted in tremendous interest in cross-layer optimization [43, 26] for scheduling transmissions over wireless fading channels. In much of the current literature, emphasis has been mostly on developing algorithms for efficient downlink data exchange in cellular networks. Almost all of them implicitly require the presence of a centralized scheduler to achieve the desired performance improvement.

In the uplink direction, due to possibility of collisions, we need to design *optimal transmission policies* for each user so that the overall network throughput is maximized. There exists a rich literature (see for e.g. [7]) that deals with such design. However, it has been only recently [52, 38, 55] that the problem has been studied from a cross-layer perspective. The important factors that need to be considered while designing cross-layer techniques for decentralized networks are:

- *Maximum achievable throughput*: As mentioned before, in decentralized networks each user wishes to maximize its own throughput. Since lack of coordination can lead to zero overall throughput, each user must be programmed with a predetermined transmission policy to ensure that the *overall* network throughput is maximized.

- *Channel estimation error*: Since cross-layer based transmission protocols usually operate depending on instantaneous channel conditions of the individual users, it is imperative to study the network performance under estimation errors. This occurs often in practice since any channel measurement is bound to be corrupted by noise leading to imperfect *channel state information* (CSI).

- *Stability of queues*: Current wireless networks are highly heterogenous catering to diverse QoS requirements of various users. It is then necessary to consider the problem of buffer overflow while designing a cross-layer based transmission policy. Buffer overflow can be avoided if the incoming packet rate from the higher layer is less than the maximum achievable throughput of the network. Also, the stability region of a random access protocol mainly determines the class of scheduling algorithms that can be applied to improve the network performance.

Almost all the existing literature that deals with cross-layer design for decentralized networks (see for e.g. [38, 55, 52]) deals with single carrier networks. In our research, we describe generic random access schemes suited for multicarrier networks. We then study the effect of multicarrier diversity on the maximum achievable throughput of the proposed scheme. We do so by investigating the effect of imperfect CSI on the throughput performance and the effect of carrier selection on the achievable rate regions.

This thesis is organized as follows: In Chapter I, we give an overview of the existing literature on cross-layer design and random access schemes. In Chapter II, we briefly present the *channel aware multicarrier random access* (CAMCRA) scheme based on carrier selection. To emphasize the role of multicarrier diversity, we study the effect of imperfect CSI on the performance of a well-known random access scheme - channel aware Aloha and derive bounds on the maximum achievable throughput. Following that, in Chapter III, we consider the stability region of two-user Aloha networks with multiple carriers. In particular, we compare our result with the classical result of [4]. Finally, in Chapter IV, we consider Aloha networks with capture and study the stability properties with the CAMCRA scheme and the NS-RA scheme.

1.1 Literature Review

1.1.1 Cross-Layer Design

Many of the existing cross-layer methods of scheduling have been designed for the downlink transmission of packets and assumes that there exists a centralized scheduler to co-ordinate the transmission process [51, 26, 33]. For example, depending on the channel state feedback from the users to the base station (BS), [33] exploits the multiuser diversity present in the wireless fading channels to improve network performance. In [24], the problem of resource allocation for downlink OFDM networks has been investigated in considerable detail from a cross-layer perspective. In [18, 19], utility function based cross-layer optimization techniques have been proposed for OFDM networks. In almost all of the literature, multicarrier diversity has been exploited in one way or the other to achieve better throughput.

The problem of cross-layer design for uplink transmission has been recently studied in [38, 52, 53]. The capacity of wireless networks for collision channels without feedback has been derived in [20] and this places upper bounds on the amount of information that can be transmitted when employing a random access scheme. In [52], a novel decentralized MAC protocol that exploits multiuser diversity has been proposed to maximize the overall throughput. In [38], *random access* (RA) in single-carrier systems with partial *channel state information* (CSI), where each user has access only to its own channel conditions, is treated comprehensively. It has been shown in [38] that population dependent transmission control can lead to significant increase in the maximum achievable throughput of the network. In [28], the problem of finding the optimal transmission policy for decentralized networks employing channel aware random access schemes has been thoroughly investigated. In particular, it has been shown in [28] that the optimal transmission policy for most decentralized networks is of a simple ON-OFF type for a large class of fading models. However, the channel model assumed in [28] has some drawbacks and hence the validity of the results need to be strengthened under more realistic assumptions.

1.1.2 Imperfect CSI

In much of the literature on design of decentralized MAC protocols, it is assumed that perfect channel state information (CSI) is available for every user. Moreover, it is assumed that channel statistics can be estimated with perfect accuracy. This is unrealistic since there is always some error in estimating the CSI and hence the channel statistics. In [27], the effect of utilizing imperfect CSI on the capacity of a time-varying wireless channel has been investigated. The rate of time variation of the channel has been shown to have a direct correlation with the loss in mutual information due to imperfect channel knowledge. In practical cellular environments, we expect the throughput to be drastically reduced due to fast-fading. However, this is not so. In [8], it has been shown that multiuser diversity in cellular networks can be exploited effectively to reduce the feedback needs while preserving the essential of the scheme performance. Recently, in [41, 42], a 1-bit feedback scheme has been proposed for downlink scheduling in OFDM networks that asymptotically achieves the same throughput as a scheme with perfect knowledge of the CSI [33, 19]. Thus, it has been shown that very coarse knowledge of CSI is sufficient to guarantee asymptotically optimal performance in the downlink of OFDM networks.

The problem of random access schemes operating with imperfect CSI has not been given much attention to. In [32], sensitivity of network performance to errors in channel parameters is discussed for opportunistic Aloha. The focus is mainly on shaping the underlying channel state distribution to enhance the throughput of the network. This requires additional complexity at the transmitter as it has to adapt its access control mechanism to achieve the target distribution *a priori* [38]. We then investigate the effect of imperfect CSI on the performance of uplink networks employing channel aware Aloha. We specifically consider the channel aware Aloha scheme [38, 52] to show the effect of imperfect CSI on the throughput of uplink networks.

1.1.3 Stability of Queues

Stability of two user Aloha networks was first discussed in [4] in the context of single packet reception networks. Following that, [40, 39] have given a complete characterization of infinite user Aloha networks equipped with multipacket reception. Recently, the stability region for two user Aloha networks with *multipacket reception* (MPR) has been completely characterized in [46]. Partial results have also been obtained for a general N user network.

For a general N user Aloha network, queuing analysis seems to be much more difficult with only the inner and outer bounds available for the stability region. In [36], the principle of stochastic dominance has been used to find inner bounds on the stability region for the general N user Aloha network. In [50], necessary and sufficient conditions for the stability of queues for a fixed transmission probability vector for the N user Aloha network has been determined. Recently, tighter inner and outer bounds on the stability region has been determined in [49]. We consider the effect of multicarrier diversity on the achievable stability region of two user Aloha networks. Specifically, we allow carrier selection based on instantaneous channel gains to improve upon the achievable throughput.

1.2 Impact and Future Work

To evaluate the practicality of the proposed work, we appeal to the current wireless standards that are ever evolving to keep up with the increasing demand for higher QoS. In IEEE 802.16a (see for e.g. http://www.ieee802.org/16/pubs/80216a-2003.html), a multicarrier network is used for data reservation by the mobile users. By the application of our protocol, it is possible to improve the data rate in such networks.

Another potential application of the proposed work is in the emerging field of *cognitive radios* [10, 31]. Cognitive radios (CRs) are smart radios designed to adapt itself to the surrounding environment and perform functions that best serve the user. The CRs are unlicensed occupying licensed bands like, for e.g., TV bands with the understanding that they vacate upon arrival of the licensed user. Thus, they essentially scavenge for white holes in the available spectrum for data transmission. *Orthogonal frequency division multiplexing* (OFDM) is naturally suitable for such networks and this allows for the application of our proposed multicarrier scheme. Another crucial feature is that the bandwidth available to the CR network varies with time and this results often in reduced throughput. With the proposed scheme, however, it is possible to exploit multicarrier diversity to offset much of the performance degradation.

Throughout our work, we have considered the case of error in channel estimate and its related statistics. A deeper problem would be to analyze the effect of model mismatch and its subsequent degradation in throughput. It is of primary interest to understand the degree of sensitivity of the random access networks and consider the effect of model mismatch on the underlying channel state distribution. Very often, in the literature [46, 33, 19], it is assumed that the channel statistics are Rayleigh distributed to simplify the analysis and lend insight into the working of protocols. However, in practice, channels could experience the generalized Nakagami fading of which Rayleigh fading is a particular example. It is therefore important to understand the effect of model mismatch on the throughput performance of random access networks. As done in our previous research, we will consider the special case of channel aware Aloha to illustrate our results.

Another issue to consider is the stability region for general N user networks. So far, in our research, we have considered the simple two user network to characterize the stability region. For future work, we will attempt to derive the stability region for a general N user asymmetric Aloha network. This is important because with the ever growing demand, future wireless networks will have stringent delay deadlines and an accurate knowledge of the stability region will ensure that the network design results in as minimal delay as possible. Also, we would like to incorporate channel uncertainties to see how the stability region is affected. More specifically, we will derive the stability region of multicarrier channel aware Aloha with imperfect channel conditions.

CHAPTER II

CHANNEL AWARE ALOHA WITH IMPERFECT CSI

2.1 Introduction

Currently, in the network layered architecture, each layer performs operations independent of the other layers to keep the design simple. This has resulted in tremendous interest in cross-layer optimization [43, 26] for scheduling transmissions over wireless fading channels.

Most of the current literature on cross-layer methods of scheduling is concerned with the downlink transmission of packets and assumes that there exists a centralized scheduler to co-ordinate the transmission process [51, 26, 33]. Assuming that the channel state feedback from the users to the base station (BS) is perfect, [33] explicitly exploits the multiuser diversity present in the wireless fading channels to improve network performance. The problem of cross-layer design for uplink transmission has been treated recently in [38, 52, 53]. The capacity of wireless system with an underlying collision channel for systems without channel feedback has been derived in [20]; this places limits on the amount of information that can be transmitted using a random access scheme. In [38], *random access* (RA) in single-carrier systems with partial *channel state information* (CSI), where each user has access only to its own channel conditions, is treated comprehensively. It has been shown in [38] that *population dependent transmission control* (PDTC) can lead to significant increase in the *asymptotic stable throughput* (AST) of the system. In [52], a channel aware Aloha scheme has been proposed for Rayleigh fading networks with each user transmitting only if its channel condition is higher than a predetermined threshold; the threshold is chosen to maximize the probability of success. In [14], we have proposed a generic multicarrier random access scheme, which exploits multicarrier diversity, and have shown increase in throughput by as much as 50%.

In all the above work, it is assumed that perfect CSI is available for every user. Moreover, it is assumed that channel statistics can be estimated with perfect accuracy. This is unrealistic since there is always some error in estimating CSI and channel statistics. Recently, [41] proposed a 1-bit feedback scheme for downlink scheduling in OFDM networks that asymptotically achieves the same throughput as a scheme with perfect knowledge of the CSI [33, 19]. Thus, it has been shown that very coarse knowledge of CSI is sufficient to guarantee asymptotically optimal performance in the downlink of OFDM networks.

In this chapter, we investigate the effect of imperfect CSI on the performance of uplink networks employing channel aware Aloha. We specifically consider the channel aware Aloha scheme [38, 52] to show the effect of imperfect CSI on the throughput of OFDM uplink networks. In [32], sensitivity of network performance to errors in channel parameters is discussed for opportunistic Aloha. However, the underlying CDMA PHY layer is assumed to be capable of multipacket reception. In contrast, we assume a multicarrier network with orthogonal carriers as the underlying PHY layer and a more strict collision model [7]. Since the collision model does not allow multipacket reception, this significantly alters the random access protocol and as a result, the maximum achievable throughput of the network. Moreover, in [38] and [32], the focus is mainly on shaping the underlying channel state distribution to enhance the throughput of the network. This requires additional complexity at the transmitter as it has to adapt its access control mechanism to achieve the target distribution *a priori* [38]. We propose the use multichannel networks and exploit the inherent diversity gain as an *alternative* to the above mentioned approach. Here the *a posteriori* channel distribution is shaped due to the use of a threshold and the presence of multiple carriers. Our protocol can be incorporated into existing random access schemes with minimal changes.

Our main result is that uncertainty in channel conditions, however small it may be, leads to *zero* throughput asymptotically for single carrier networks with arbitrarily large number of users [12, 15]. In other words, single carrier networks employing the Aloha scheme have an average asymptotic throughput of zero under uncertain channel conditions. However, this is not so in multicarrier networks operating under the same bandwidth. We prove that the effect of improper knowledge of CSI can be offset by exploiting the inherent diversity in multicarrier networks. Throughout the chapter, we assume that the transmit power is fixed for all users under consideration.

The chapter is organised as follows: In Section 2.2, we present the channel model we will be using throughout the chapter. In Section 2.3, we present the *channel aware multicarrier random access* (CAMCRA) [14] and the *no selection-random access* NS-RA schemes. In Sections 2.4 and 2.5, we study the success probability in the presence of channel estimation errors for the NS-RA and the CAMCRA schemes, respectively. Apart from that, in Section 2.6, we consider the overall network throughput performance of the NS-RA and the CAMCRA schemes. In Section 2.7, we discuss certain practical issues that need to be considered for the CAMCRA scheme. Finally, in Section 2.8, we present our conclusion.

2.2 Channel Model

In this chapter, we assume that all users experience identical Rayleigh fading. Moreover, channel fades corresponding to different users are assumed to be independent. If a signal x is sent, the received signal y is given by

$$y = hx + w,$$

where the fading coefficient h and the additive noise w are modelled as independent zero-mean unit variance complex Gaussian random variables. We also assume that each user has access to its channel state information. This is facilitated by allowing pilot symbols to be transmitted at regular intervals [18].

Given the *true* channel fading coefficient h, the estimate of the channel $\hat{h}$, as obtained by the user is given by

$$\hat{h} = h + e, \tag{2.1}$$

where e is the error in the estimate. We assume that the estimation error e is complex Gaussian [54] (possibly dependent on h) with zero mean and variance σ_e^2. In any channel aware random access scheme, the user makes decisions about packet transmission based on the estimated channel coefficient $\hat{h}$. Further, let $\sigma_{act}^2 = E\{|\hat{h}|^2\}$ denote the actual variance of $\hat{h}$. In practice, σ_{act}^2 is never known perfectly to the user and has to be estimated from $\hat{h}$

itself. Let σ_{est}^2 denote the *estimated* variance of $\hat{h}$ and define

$$\omega = 1 - \frac{\sigma_{est}^2}{\sigma_{act}^2} \tag{2.2}$$

to be an uncertainty parameter. In literature, the *minimum mean squared error* (MMSE) estimator [37, 47] is also employed for channel estimation purposes which assumes knowledge of the channel statistics; in particular the channel variance σ_{act}^2. In this chapter, however, we have not restricted ourselves to any particular class of estimators. As a result, $\sigma_{est}^2 \in (0, \infty)$ could take any positive value implying that $-\infty < \omega < 1$. We can see that $\omega = 0$ refers to perfect knowledge of the CSI. As we shall show in Section 2.6, this parameter is crucial in determining the asymptotic throughput of random access networks operating with imperfect CSI.

It is important to note that the users themselves are unaware of the uncertainty ω since the error in estimating the channel and the related statistics are usually unknown. If ω were known to the users, then it is equivalent to the case of knowledge of perfect CSI.

2.3 Channel Aware Multicarrier Random Access Scheme

In this section, we describe a generic random access scheme for a network with multiple carriers. This is achieved by making a simple modification to any existing single carrier random access scheme [14].

Consider a network with N_u users and N_c carriers. Let the set of users be represented as $\mathcal{U} = \{1, 2, ...N_u\}$ with $|\mathcal{U}| = N_u$. We assume that the time axis is divided into slots of equal length and packet transmission occurs at the beginning of every slot. Let $\mathcal{A}_{\mathcal{SC}}(N_u)$ denote any distributed single carrier random access scheme with N_u users. Also, for the i^{th} user, let $\hat{h}_{i,k}$ denote the estimated channel fading coefficient of the k^{th} channel. We then describe the *channel aware multicarrier random access* (CAMCRA) scheme as follows:

CAMCRA scheme: In each slot s,

1. *User i, $1 \leq i \leq N_u$, chooses carrier $j(i)$, where*

$$j(i) = \arg\max_{1 \leq k \leq N_c} |\hat{h}_{i,k}|^2. \tag{2.3}$$

Let $\mathcal{U}_j \subseteq \mathcal{U}$, represent the set of users competing for the j^{th} carrier and let $n_j = |\mathcal{U}_j|$.

2. *In each carrier $j, 1 \leq j \leq N_c$, apply the $\mathcal{A}_{SC}(n_j)$ scheme.*

In the above description, $\arg\max$ denotes the carrier index with the maximum gain. Thus in the CAMCRA scheme, each user is allowed to choose the carrier with the best gain. This intuitively reduces the contention per carrier and also exploits multicarrier diversity. As we shall prove later, this indeed affects the stability properties of random access collision networks. Following the above notation, we have the trivial random access scheme for a network with N_u users and N_c carriers:

No Selection - Random Access (NS-RA):

In each carrier j, with $1 \leq j \leq N_c$, apply the $\mathcal{A}_{SC}(N_u)$ scheme.

It must be emphasized at this point that the proposed CAMCRA scheme is generic and is applicable to any existing random access scheme. The main idea of this chapter is to illustrate the improved stability properties of multicarrier random access networks by choosing carriers judiciously. Below, we have considered a particular random access scheme, the channel-aware Aloha, and studied the stability properties with (CAMCRA) and without (NS-RA) carrier selection.

Before we present our results, we briefly summarize the assumptions we use in our analysis.

- The transmission is slotted.
- The fading process is independent from one time slot to another and also from one carrier to another.
- The total transmit power for each user is restricted to be P_{tot}.
- The total bandwidth available to the network is B Hz.

We now investigate the performance of multicarrier networks under the NS-RA and CAMCRA schemes under imperfect CSI. Two common measures that determine the performance of any random access scheme are: (a) probability of success per carrier and (b) overall network throughput. In Section 2.4 and Section 2.5, we discuss the probability of success per carrier with *imperfect* CSI under NS-RA and CAMCRA schemes, respectively. In Section 2.6, we discuss the impact on overall network throughput.

2.4 Multicarrier Networks With NS-RA

In this section, we investigate the effect of imperfect CSI on the performance of multicarrier random access networks employing the NS-RA scheme described in Section 2.3. First, we describe the template random access scheme that will be used throughput the chapter to illustrate the power of CAMCRA over NS-RA.

2.4.1 Channel Aware Aloha

In this section, we briefly discuss the *channel-aware Aloha* (CAA) scheme [38, 52] that has been proposed for decentralized single carrier wireless networks. We use the CAA as the template $\mathcal{A}_{SC}(N_u)$ for performance comparison of the CAMCRA and NS-RA schemes described above. Also, we consider here the classical collision model [7] under slotted transmission. Let $\mathcal{U} = \{1, 2, ...N_u\}$ denote the set of users and let $\hat{h}_i$ denote the estimated instantaneous channel coefficient of the i^{th} user.

Channel-Aware Aloha (CAA) [38, 52]: *In each slot s*

1. Each user chooses a threshold $H_0(N_u)$. Let $\mathcal{U}_0 = \{i : |\hat{h}_i|^2 > H_0(N_u)\}$.

2. If $|\mathcal{U}_0| = 1$, then *success*, if $|\mathcal{U}_0| = 0$ *idle*, else *collision.*

From the above description, it is easy to see that in CAA all the N_u users in the network contend for the available carrier. Moreover, each user decides to transmit if and only if its instantaneous gain is greater than a particular threshold H_0. Various other transmission control schemes have appeared in the literature [38, 55] that are in general non-trivial functions of the estimated channel coefficients. In this chapter, we analyze the thresholding scheme presented above mainly from the point of view of ease of implementation. Let a_{N_u} denote the probability of success of the CAA scheme when N_u users contend for the available carrier. We recall that in the collision model, a packet is successfully transmitted if and only if there is no other packet transmitted in the same slot. If each user transmits in a single slot with a probability p_{N_u}, then it is easy to see that [7]

$$a_{N_u} = N_u p_{N_u} (1 - p_{N_u})^{N_u - 1}.$$

It is also easy to show [38][7] that choosing the access probability $p_{N_u} = \frac{1}{N_u}$ maximizes the overall probability of success to

$$a_{N_u} = \left(1 - \frac{1}{N_u}\right)^{N_u - 1}. \tag{2.4}$$

Suppose now that each user i has perfect knowledge of its instantaneous fading coefficient h_i. For a given threshold H_0, the access probability is given by $p_{N_u} = \Pr\{|h_i|^2 > H_0\}$. Since the users experience Rayleigh fading with $E\{|h_i|^2\} = 1$, we find that $p_{N_u} = \Pr\{|h_i|^2 > H_0\} = e^{-H_0}$. To achieve maximum overall success probability, it then follows that $e^{-H_0} = \frac{1}{N_u}$ or

$$H_0(N_u) = \ln N_u. \tag{2.5}$$

Thus to determine H_0, it is necessary to know the total population of the network N_u. In the above analysis, we allow all the users to undergo identical fading/shadowing, and hence the threshold H_0 needs to be the same for all the users in order to maximize the overall success probability. If on the other hand, the users undergo non-identical fading, then we note that the threshold H_0 can be adjusted accordingly for each user to ensure that $\Pr\{H_i > H_{0,i}\} = \frac{1}{N_u}$.

Suppose now that the CSI is not known exactly as explained in (2.1) and (2.2). In other words, for a true channel coefficient of h_i, the channel as estimated by the user is $\hat{h}_i = h_i + e_i$ where the estimation error e_i is modelled as a zero-mean complex Gaussian random variable (possibly dependent on h_i) with variance σ_e^2. Further, $\omega = 1 - \frac{\sigma_{est}^2}{\sigma_{act}^2}$ denotes the uncertainty parameter where σ_{est}^2 and σ_{act}^2 denote the estimated and actual variances of the channel coefficient $\hat{h}$. Since the channel coefficient estimated by the user i is $\hat{h}_i$, it can be seen that the access probability for a threshold $\hat{H}_0$, is given by $\Pr\{|\hat{h}_i|^2 > \hat{H}_0\} = e^{-\frac{\hat{H}_0}{\sigma_{act}^2}}$.

Since the estimated variance of the channel fading is σ_{est}^2, the threshold chosen by the user can be determined as

$$\hat{H}_0(N_u) = \sigma_{est}^2 \ln N_u.$$

In this case, the probability $\hat{p}_{N_u}$ that user i transmits in a particular slot is given by

$$\hat{p}_{N_u} = \Pr\{|\hat{h}_i|^2 > \hat{H}_0(N_u)\} = e^{-\frac{\hat{H}_0(N_u)}{\sigma^2_{act}}} = \frac{1}{{N_u}^{1-\omega}} \tag{2.6}$$

and the overall probability of success is

$$a_{N_u}(\omega) = N_u \hat{p}_{N_u}(1-\hat{p}_{N_u})^{N_u-1} = {N_u}^{\omega}\left(1 - \frac{1}{{N_u}^{1-\omega}}\right)^{N_u-1}, \tag{2.7}$$

where the uncertainty ω is given by (2.2) with $-\infty < \omega < 1$. When the CSI is perfectly known ($\omega = 0$), the expression for a_{N_u} in (2.7) reduces to that in (2.4).

2.4.2 NS-RA with Imperfect CSI

Consider now a multicarrier network with N_c carriers and N_u users following the NS-RA scheme described in Section 2.3. As in [14], we define the population density $\alpha = \frac{N_u}{N_c}$ and consider success probability and network throughput in terms of the population density α. To determine asymptotic success probabilities, we let $N_u, N_c \to \infty$ keeping α fixed [14]. Let $\zeta_{sc}(\alpha, \omega)$ denote the average probability of success for any network following the NS-RA scheme with an uncertainty ω. We recall that in the NS-RA scheme, in each carrier, all the N_u users contend for any particular channel. Therefore, from (2.7), we have that

$$\zeta_{sc}(\alpha, \omega) = {N_u}^{\omega}\left(1 - \frac{1}{{N_u}^{1-\omega}}\right)^{N_u-1}. \tag{2.8}$$

When there is perfect CSI ($\omega = 0$), it is well known [7] that $\zeta_{sc}(\alpha, 0) \to e^{-1}$ as $N_u \to \infty$. The quantity e^{-1} denotes the asymptotic success probability of the channel aware Aloha scheme with perfect CSI. It is important to study asymptotic success probability for the following reason. Consider a network of N_u users each equipped with a buffer capable of holding arbitrarily large number of packets. Further, let the arrival process of each user be an independent Poisson process with an arrival rate of $\frac{\lambda}{N_u}$, so that the overall data rate into the network is λ. Then it is a well known fact [7] that the buffer of each user is guaranteed to be stable (see [50] for precise definitions) only if $\lambda < e^{-1}$. Thus the asymptotic success probability also determines the maximum allowable input rate of the network.

For all non-zero ω, we have the following proposition regarding the asymptotic success probability (see Appendix A).

Proposition 1 *Consider a multicarrier network with N_c carriers and N_u users following the NS-RA scheme under uncertainty ω. For any network density $\alpha = \frac{N_u}{N_c}$, let $\zeta_{sc}(\alpha, \omega)$ given by (2.8) be the probability of success per carrier. Then as $N_u, N_c \to \infty$ with α fixed,*

$$\zeta_{sc}(\alpha, \omega) \to 0$$

as $N_u \to \infty$, for all $\omega \neq 0$. $\square$

Thus any degree of uncertainty in the channel estimate, however small it may be, leads to zero asymptotic success probability in networks following the NS-RA scheme. Consequently, the maximum allowable input rate in NS-RA networks operating under uncertain channel conditions is zero!

It is important to note that the result in Proposition 1 holds even if the channel estimate h is perfect, but the user does not know the variance, $E\{|h|^2\}$, accurately. We conclude that the performance of the NS-RA scheme degrades with the number of users when there is uncertainty in channel parameters. Intuitively, the explanation behind such drastic performance is this: when there is perfect CSI ($\omega = 0$), the threshold chosen by each user is $H_0 = \ln N_u$ or, equivalently, each user "assumes" correctly that there are N_u users in the network and chooses the probability of transmission to be $\frac{1}{N_u}$. However, when $\omega \neq 0$, the "equivalent" threshold chosen by each user is $\ln({N_u}^{1-\omega})$ or each user "assumes" that there are ${N_u}^{1-\omega}$ users in the network and chooses the transmission probability accordingly. When $\omega < 0$, each user assumes that there are more users than the actual number and reduces its channel access probability leading to reduced throughput. When $0 < \omega < 1$, each user assumes that there are fewer number of users than the actual and tries to access the channel more frequently leading to increased collisions and reduced throughput.

From the above discussion, it is obvious that we can roughly translate the error in channel estimate to error in network size estimate. It must be then noted that NS-RA shows such drastic performance only because the error in network size *exponentially* varies with the network size. This is in contrast to the results reported in [32] where the error in network size is modelled as a Poisson random variable with mean equal to the actual network size. Typically, the number of users in the network may vary randomly over a certain period of time, and hence it may be hard to estimate the network population from slot to slot. Hence most protocols assume a fixed number of users in order to determine the access probability. In [32], the network size $\hat{N}_u$ is modelled as a Poisson random variable with mean equal to the assumed network size N_u. In other words, although the actual network size varies randomly as $\hat{N}_u$, each user accesses the channel with a probability $\frac{1}{N_u}$. It is not hard to show that the expected probability of success under the collision model would then be

$$E\left\{\frac{\hat{N}_u}{N_u}\left(1-\frac{1}{N_u}\right)^{\hat{N}_u-1}\right\} = \frac{e^{-1}}{1-e^{-N_u}}$$

which converges to e^{-1} as $N_u \to \infty$. In general, if N_u denotes the true network population, assuming that the number of users is $\hat{N}_u = aN_u + b$ where $a, b < \infty$ does not affect the asymptotic throughput. However, assuming $\hat{N}_u = N_u^c$ with $c \neq 1$ does lead to asymptotic zero throughput.

2.5 CAMCRA with Imperfect CSI

In this section, we determine the overall success probability of CAMCRA scheme with imperfect CSI. Intuitively, we expect that the CAMCRA scheme should fare better than NS-RA. The reason is that in the NS-RA scheme, all the N_u users contend in each carrier. However in the CAMCRA scheme, since the carriers for each user are i.i.d., the probability that a particular carrier j is chosen for transmission is given by $p_t = \frac{1}{N_c}$ where N_c refers to the total number of carriers. If n_j denote the number of users contending for the j^{th} carrier it can be seen that

$$\Pr\{n_j = k\} = \binom{N_u}{k}\left(\frac{1}{N_c}\right)^k\left(1-\frac{1}{N_c}\right)^{N_u-k} \tag{2.9}$$

It follows that the average number of users contending per carrier in the CAMCRA scheme is $E\{n_j\} = \frac{N_u}{N_c} = \alpha$. Thus, for example, in a network of $N_u = 100$ users and $N_c = 20$ carriers operating with uncertainty $\omega = 0.3$, the average number of users contending per carrier is 5 in CAMCRA but 100 in NS-RA. Correspondingly, from (2.7), the overall success probability in the case of NS-RA would be $(100)^{0.3}\left(1-\frac{1}{(100)^{0.7}}\right)^{199} \approx 0.071$. To determine

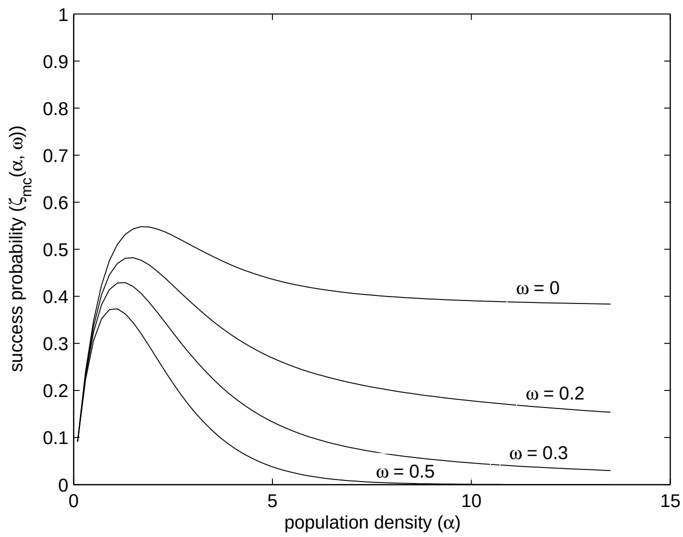

Figure 1: Asymptotic success probability of the CAMCRA scheme.

the corresponding success probability for the CAMCRA scheme, we need the following proposition (see Appendix A).

Proposition 2 *Consider a multicarrier network with N_c carriers and N_u users following the CAMCRA scheme under uncertainty ω. For any network density $\alpha = \frac{N_u}{N_c}$, let $\zeta_{mc}(\alpha, \omega)$ be the probability of success per carrier. Then as $N_u, N_c \to \infty$ with α fixed,*

$$\zeta_{mc}(\alpha, \omega) \to e^{-\alpha} \sum_{k=1}^{\infty} \frac{\alpha^k}{k!} a'_k(\omega), \tag{2.10}$$

where $a'_n(\omega)$ is the success probability per carrier given that n users contend for a particular carrier and satisfies $\lim_{n\to\infty} a'_n(\omega) = 0.\square$

The exact expression for $a'_n(\omega)$ is derived in Appendix A. In Figure 1, we have numerically evaluated the probability of the CAMCRA scheme given by (2.10) and plotted it as a function of the network density α for various values of the uncertainty parameter ω. In particular, for $\alpha = 5$ and $\omega = 0.3$, we see that the success probability per carrier due to the CAMCRA scheme is approximately 0.15, nearly double that of 0.071 evaluated earlier for NS-RA.

As a final note, we would like to mention that the results obtained so far are valid when the carriers are chosen randomly instead of choosing the carrier with the best gain.

The reason is that when the carrier is chosen randomly by the user, multicarrier diversity is not exploited and the only advantage is from the reduced population per carrier. Also, the above results are valid if the users employ some kind of power control mechanism to ensure a target received SNR since the transmission rate of each user is then fixed and the throughput is determined by the probability of success.

2.6 Network Throughput

So far, we have discussed extensively the probability of success of the NS-RA and the CAMCRA schemes in a single slot with the channel aware Aloha as the template random access scheme. However, in practice, we are more concerned about the overall throughput of the network. In particular, we are concerned about the maximum throughput that can be supported by the NS-RA and the CAMCRA scheme without causing buffer overflow. This has been termed as the maximum stable throughput in [38]. Consider a network with N_c carriers following the NS-RA scheme described in Section 2.3 in which every carrier has N_u contenders. Let $\mathcal{U} = \{1, 2, ..., N_u\}$ denote the set of users. If each user has access to its *perfect* CSI $\{h_{i,j}\}_{i=1,j=1}^{N_u,N_c}$ then given that a particular slot resulted in a successful transmission for a particular carrier j, the throughput achieved is $\frac{1}{N_c}R(\max_{1\leq i\leq N_u}|h_{i,j}|^2)$, where $R(|h|^2)$ denotes the achievable rate when the channel fading coefficient is h. If we assume the presence of capacity achieving codes, then [44]

$$R(|h|^2) = B\log_2\left(1 + \frac{P_{tot}}{BN_0}|h|^2\right) \tag{2.11}$$

where, P_{tot} is the total transmit power, N_0 is the noise power, and B denotes the total available bandwidth. Without loss of generality, we let $\frac{P_{tot}}{N_0} = B = 1$.

In the channel aware Aloha with imperfect CSI, we note from the description in Section 2.4 that each user transmits in a particular carrier if and only if its threshold is greater than a predetermined threshold $\hat{H}_0$. When the CSI is imperfect, let the channel estimate be $\hat{h}_{i,j} = h_{i,j} + e_{i,j}$ as given by (2.1) with $e_{i,j}$ modelling the CSI measurement error and the uncertainty parameter ω be given by (2.2). It follows that the individual users base the transmission decisions on erroneous channel estimates and hence the maximum achievable throughput in carrier j is $\frac{\hat{R}}{N_c}$ where

$$\hat{R} = R(|\hat{h}_{m(j),j}|^2), \tag{2.12}$$

with $m(j) = \arg\max_{1 \le i \le N_u} \{|h_{i,j} + e_{i,j}|^2\}$.

In the NS-RA scheme, since all the N_u users contend in each carrier, we find that the overall throughput is obtained as

$$\eta_{sc}(\alpha, \omega) = E\left\{\sum_{j=1}^{N_c} \frac{1}{N_c} R(|\hat{h}_{m(j),j}|^2)\mathbb{1}(E_j)\right\}, \tag{2.13}$$

where $\mathbb{1}(.)$ denotes the indicator function and E_j denotes the event

$$\{|\hat{h}_{m(j),j}|^2 > \hat{H}_0\} \bigcap_{i \neq m(j)} \{|\hat{h}_{i,j}|^2 < \hat{H}_0\}.$$

The above equation can be simplified as

$$\eta_{sc}(\alpha, \omega) = N_u(1 - \hat{p}_{N_u})^{N_u - 1} E\{R(|\hat{h}_{m(1),1}|^2)\mathbb{1}(|\hat{h}_{m(1),1}|^2 > \hat{H}_0)\} \tag{2.14}$$

where $\hat{p}_{N_u}$ denotes the access probability of any user in the NS-RA scheme with channel uncertainty and is given by (2.6). Note that the above expression can also be obtained from Theorem 1 of [38].

Regarding the asymptotic behaviour of $\eta_{sc}(\alpha, \omega)$, we have the following result (see Appendix A).

Proposition 3 *Consider a multicarrier network with N_c carriers and N_u users following the NS-RA scheme under uncertainty ω. For any network density $\alpha = \frac{N_u}{N_c}$, let $\eta_{sc}(\alpha, \omega)$ defined by (2.14) be the overall average throughput. Then as $N_u, N_c \to \infty$ with α fixed, we have*

$$\eta_{sc}(\alpha, 0) = \log_2(1 + \ln N_u)\zeta_{sc}(\alpha, 0) + o(1) \tag{2.15}$$

and for any $\omega \neq 0$,

$$\eta_{sc}(\alpha, \omega) \to 0, \tag{2.16}$$

where $\zeta(.,.)$ is as described in (2.8). $\square$

Thus we find that any uncertainty in channel conditions, however small it may be, leads to *zero* asymptotic throughput in the NS-RA scheme. We have already seen that the probability of success does not asymptotically go to zero in case of the CAMCRA scheme. We would like to see the asymptotic network throughput behaviour of the CAMCRA scheme. The important question now is, does the asymptotic throughput of the CAMCRA scheme go to zero or is it bounded away from zero? For that we need the following important result which is also of independent interest.

Multiuser Diversity Theorem (MDT) *Consider a slotted network of N_u users, $1 \leq i \leq N_u$, with h_i and $\hat{h}_i = h_i + e_i$ denoting the instantaneous exact and estimated channel coefficients, respectively, of the i^{th} user. The maximum achievable throughput is given by*

$$\hat{R} = \log_2(1 + |h_M|^2), \tag{2.17}$$

where $M = \arg\max_{1 \leq i \leq N_u}\{|h_i + e_i|^2\}$. Then we have

$$E\{\hat{R}\} \geq \log_2(1 + A^2 \ln N_u) - o(1) \tag{2.18}$$

where $A = \sigma_{act} - \sigma_e$, $E\{|h_i|^2\} = 1, E\{|\hat{h}_i|^2\} = \sigma_{act}^2$, and $E\{|e_i|^2\} = \sigma_e^2$, respectively. □
The proof can be found in Appendix A.

It is well known that (see for e.g. [18]) when there is perfect CSI ($e_k = 0$), we have $E\{\hat{R}\} = \log_2(1 + \ln N_u) + o(1)$. Therefore in the presence of channel uncertainties, we must have $E\{\hat{R}\} \leq \log_2(1 + \ln N_u) + o(1) = O(\ln \ln N_u)$. The above theorem essentially asserts the the lower bound is also of the order of $\ln \ln N_u$. It can be shown that $|A| \leq 1$[1] and as a result, the lower bound (2.18) is consistent with the upper bound. In essence, we find that multiuser diversity is very effective in offsetting the degradation in performance due to the uncertainty in channel estimate. From MDT, we see that the net throughout is still of the order of $\ln \ln N_u$, no matter how uncertain we are about the channel estimate.

We would like to know whether the CAMCRA scheme also ensures non-zero asymptotic throughput under uncertainty in CSI. If so, we would like to know what exactly is the loss

[1] From (2.1), $\sigma_{act}^2 = E\{|h+e|^2\} = E\{|h|^2\} + E\{|e|^2\} + 2ReE\{he^*\}$. Since $E\{|h|^2\} = 1, E\{|e|^2\} = \sigma_e^2$, and $|ReE\{he^*\}|^2 \leq |E\{he^*\}|^2 \leq E\{|h|^2\}E\{|e|^2\}$, we obtain that $(1-\sigma_e)^2 \leq \sigma_{act}^2 \leq (1+\sigma_e)^2$. Thus $|\sigma_{act} - \sigma_e| \leq 1$.

in throughput due to channel uncertainty. For a population density α, let $\eta_{mc}(\alpha, \omega)$ be defined as the throughput of the CAMCRA scheme operating with an uncertainty ω. We have the following result (see Appendix A).

Proposition 4 *Consider a multicarrier network with N_c carriers and N_u users following the CAMCRA scheme under uncertainty ω. For any network density $\alpha = \frac{N_u}{N_c}$, let $\eta_{mc}(\alpha, \omega)$ defined by (2.14) be the overall average throughput. Then as $N_u, N_c \to \infty$ with α fixed, we have*

$$\eta_{mc}(\alpha, \omega) > \log_2 \left(1 + A^2 \ln \frac{N_u}{\alpha}\right) \zeta_{mc}(\alpha, \omega) - o(1) \tag{2.19}$$

where $\zeta_{mc}(.,.)$ is as defined in (2.10). □

Thus, the CAMCRA scheme is much more robust to uncertainty in CSI than the NS-RA scheme. We have already proved in Proposition 2 that the success probability $\zeta_{mc}(\alpha, \omega)$ is bounded away from zero. Therefore, the throughput of the CAMCRA scheme is also lower bounded by $\ln \ln N_u$ as seen from (2.19). In Figure 2, for a multicarrier network consisting of $N_c = 20$ carriers with total available bandwidth $B = 1$ KHz, we plot the throughput as a function of the network population N_u for various values of uncertainty parameter ω. We see that the CAMCRA scheme largely outperforms the NS-RA scheme in terms of average throughput for high values of uncertainty, ω, especially as the network population increases.

2.7 Practical Issues

In this section, we discuss two important practical issues related to the implementation of the CAMCRA scheme. First, we consider the problem of collision resolution and reservation in the NS-RA and the CAMCRA schemes, respectively, in the presence of imperfect CSI. Second, we consider the effect of channel correlation on the throughput of CAMCRA and the NS-RA schemes.

2.7.1 Collision Resolution in NS-RA

To improve upon the success probability and boost the overall data rate, an opportunistic algorithm has been proposed in [53] for resolving collisions in the channel aware Aloha scheme. Specifically a binary search-like algorithm is proposed to determine the user with

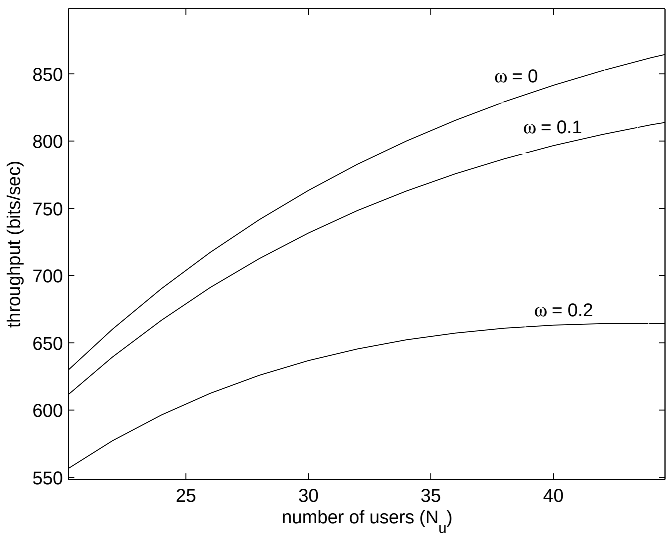

Figure 2: Effect on CSI uncertainty on the throughput of CAMCRA for various values of ω.

the best gain among all the users contending for a given carrier. This is done by allocating a small number N_t of minislots exclusively for collision resolution before transmission in every slot and it has been shown that $N_t \approx 3$ is enough to achieve success probability very close to one. We now study the performance of the splitting algorithm proposed in [53] under the presence of channel estimation errors. Let $\zeta'_{sc}(\alpha, \omega)$ denote the probability of success of the NS-RA scheme with collision resolution when the uncertainty is ω. We have the following result (see Appendix A).

Proposition 5 *Consider a multicarrier network with N_c carriers and N_u users following the NS-RA scheme under uncertainty ω. For any network density $\alpha = \frac{N_u}{N_c}$, let $\zeta'_{sc}(\alpha, \omega)$ be the average success probability per carrier with collision resolution. When there is perfect CSI ($\omega = 0$), then for any $\epsilon > 0$, there exists N_t sufficiently large that*

$$\zeta'_{sc}(\alpha, 0) > 1 - \epsilon.$$

For any $\omega \neq 0$,

$$\zeta'_{sc}(\alpha, \omega) \to 0 \tag{2.20}$$

as $N_u, N_c \to \infty$. $\square$

The opportunistic splitting algorithm is powerful when there is perfect CSI. By allowing sufficiently large time to resolve collisions, it is possible to achieve success probabilities arbitrarily close to one. However, any amount of CSI uncertainty results in asymptotic success probability of zero even with attempted collision resolution. Thus the performance of the NS-RA scheme degrades when there is uncertainty in the channel parameters.

2.7.2 Reservation in CAMCRA

In the CAMCRA scheme, we note from the description that any user contending for a particular carrier j chooses a threshold $H_0(n_j)$ which depends on the number of users n_j contending for carrier j. Thus each user in carrier j is assumed to know the exact number of contending users, n_j. This may not always be available since n_j varies from slot to slot. To circumvent this problem, we describe below, a novel and simple reservation scheme specially suited for the CAMCRA scheme.

Let $\mathcal{U}_n = \{1, 2, ..., n\}$ be the set of users contending for a particular carrier in the CAMCRA scheme. Assume that the transmission is slotted and that a small number N_t of minislots is reserved for collision resolution. Let H_0 denote any arbitrary positive number and $\{h_i\}_{i=1}^{n}$ denote the instantaneous channel fading coefficients. The reservation scheme is described below.

Reservation Scheme: In each slot s,

1. User i, $1 \leq i \leq n$, chooses an integer $r(i)$ randomly from $\{0, 1, 2, ..., N_t - 1\}$ if and only if $|h_i|^2 > H_0$.

2. User i, $1 \leq i \leq n$, transmits his request to send (RTS) packet in minislot $r(i)$ to the base-station (BS).

3. If there exists $i_0 = \min\{i : r(i) \neq r(j), 1 \leq j \leq n\}$, then user i_0 is allowed to transmit; else collision.

The above reservation scheme works as follows: Each user i contending for a particular carrier chooses a number $r(i)$ randomly between 0 and $N_t - 1$, where N_t is the number of minislots allowed for reservation. At minislot $r(i)$, the BS receives RTS from user i, if

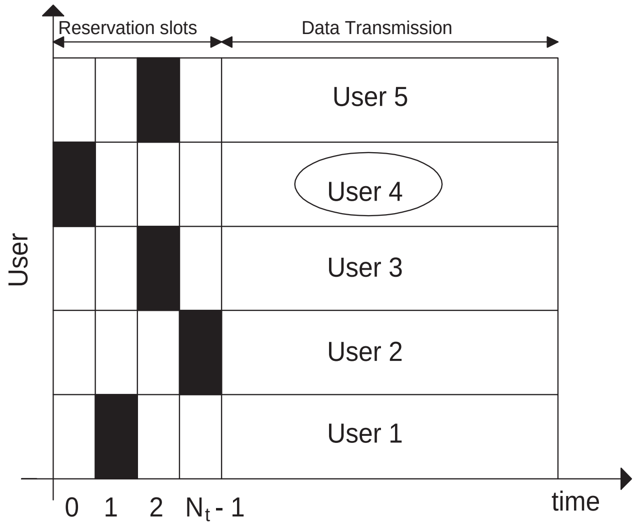

Figure 3: Reservation Scheme for CAMCRA.

and only if $r(i)$ has not been chosen by any other user. If more than one user chooses the same number $j \in \{0, 1, 2, ..., N_t - 1\}$, it results in a collision and the BS receives no RTS in minislot j. The BS sends the *clear to send* (CTS) message to that user whose RTS is received first. Figure 3 illustrates this point.

From the above description, we note that the success probability of the reservation scheme does not depend on the channel conditions. We have the following lower bound on the achievable success probability (see Appendix A).

Proposition 6 *Consider a multicarrier network with N_c carriers and N_u users following the NS-RA scheme under uncertainty ω. For any network density $\alpha = \frac{N_u}{N_c}$, let $\zeta'_{mc}(\alpha, \omega)$ denote the success probability of the CAMCRA scheme with N_t minislots allocated for reservation as described above. Then as $N_u, N_c \to \infty$,*

$$\zeta'_{mc}(\alpha, \omega) \geq \frac{\alpha}{N_t} \frac{\left(1 - e^{\frac{\alpha}{N_t} - \alpha}\right)}{\left(e^{\frac{\alpha}{N_t}} - 1\right)} - o(1). \Box$$

Thus the asymptotic success probability of the CAMCRA scheme employing reservation is bounded away from zero. We intuitively explain why the reservation scheme just described works well for the CAMCRA scheme. In the CAMCRA scheme, in any particular carrier j, the number of users n_j is binomially distributed with $E\{n_j\} = \frac{N_u}{N_c} = \alpha$. Thus, in a network with 200 users and 50 carriers, for example, an average of $\alpha = 4$ users contend per carrier. If

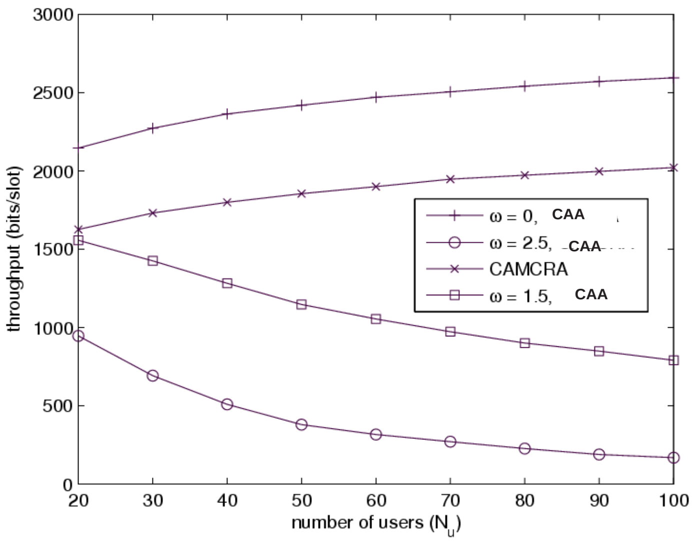

Figure 4: Performance of CAMCRA with channel reservation under imperfect CSI.

$N_t = 5$ minislots are allocated for reservation, it follows from Proposition 2 that the success probability is at least 0.626, irrespective of the channel uncertainty.

In Figure 4, we plot the throughput curves for the CAMCRA and NS-RA scheme for $\omega = 0, 1.5$, and 2.5 respectively, assuming a total bandwidth of $B = 1$ KHz. We have assumed that a total of $N_t = 5$ minislots are allotted for collision resolution for NS-RA and reservation for CAMCRA, respectively. Clearly, we see that the CAMCRA scheme outperforms the NS-RA scheme in terms of average throughput as the uncertainty increases.

2.8 Conclusion

In this chapter, we have illustrated the power of judicious carrier selection based on channel fading gains in random access multicarrier networks. To that end, we have proposed a novel channel aware multicarrier random access (CAMCRA) scheme and compared the throughput performance with the random access scheme that does not perform smart carrier selection (NS-RA). We have shown that the CAMCRA scheme offers high degree of robustness to channel uncertainties. We have then proposed a novel reservation scheme specially suited for the CAMCRA scheme to overcome possible technical implementation difficulties. We have also proved that the same order of throughput is maintained in all cases.

CHAPTER III

STABILITY REGION

3.1 Introduction

In this chapter, we investigate the effect of multicarrier diversity on the stability region of channel aware Aloha [16, 17]. The stability region for a simple two user Aloha system was first derived in [4] assuming that the packet arrival process is Poisson under the collision channel model. Recently, the stability region for Aloha networks with *multipacket reception* (MPR) has been completely characterized in [46]. We use a result from [46] to show that the stability region of multicarrier channel aware Aloha "contains" the stability region of single carrier Aloha characterized in [4]. Thus, multicarrier diversity improves the stability region of the two user Aloha network. Throughout the chapter, we assume that the transmit power is fixed for all users under consideration.

For a general N user Aloha network, queuing analysis seems to be much more difficult with only the inner and outer bounds available for the stability region. In [36], the principle of stochastic dominance has been used to find inner bounds on the stability region for the general N user Aloha network. In [50], necessary and sufficient conditions for the stability of queues for a fixed transmission probability vector for the N user Aloha network has been determined. Recently, tighter inner and outer bounds on the stability region has been determined in [49].

The chapter is organized as follows: In Section 3.2, we describe the channel model used throughout the chapter. In Section 3.3, we consider multicarrier two user networks and describe two simple schemes for the selection of carriers. We first allow the users to select the carriers randomly and derive the stability region. We then consider selection of carriers based on channel gains and show improvement in stability region because of multicarrier diversity. In Section 3.4, we present our conclusion.

3.2 Channel Model

Consider a multicarrier network of n carriers occupying a bandwidth of B Hz. Each carrier is assumed to be of bandwidth $\frac{B}{n}$ Hz. In this chapter, we assume that all carriers experience Rayleigh fading and that the transmission is slotted into intervals of equal length. Moreover, channel fades corresponding to different carriers are assumed to be independent. Strictly speaking, as the number of carriers becomes larger keeping the bandwidth constant, channel correlation comes into effect and results in lesser throughput. If a signal x is sent, the received signal y is given by

$$y = hx + w,$$

where the fading coefficient h and the additive noise w are modelled as independent zero-mean unit variance complex Gaussian random variables. We also assume that each user has access to its channel state information. This is facilitated by allowing pilot symbols to be transmitted at regular intervals [18].

Given the instantaneous channel fading coefficient h for a particular user in a particular carrier, the maximum throughput achievable in that carrier is then given by [44]

$$\frac{B}{n} E\left\{\log_2\left(1 + \frac{P_{tot}}{BN_0}|h|^2\right)\right\} \tag{3.1}$$

where, $E\{.\}$ is the expectation operator, P_{tot} is the total transmit power, and N_0 is the noise power. Throughout the chapter, we assume that the total power is uniformly distributed over all the carriers and for simplicity let $\frac{P_{tot}}{N_0} = B = 1$ and denote

$$C_{max} = E\{\log_2(1 + |h|^2)\}. \tag{3.2}$$

3.3 Multicarrier Channel Aware Aloha

Consider a network with two users U_1 and U_2 wanting to access a multichannel network with n carriers. We assume that transmission is slotted and that there are capacity achieving codes so that the throughput in any carrier and in any slot is given by (3.1). Let λ_1' and λ_2' denote the input arrival rates (in bits/sec), respectively, of the two users which are assumed to be Poisson processes independent of one another. We assume that each user has a buffer size capable of holding arbitrarily large but fixed number of packets. Also, for $i = 1, 2$, let

p_i denote the access probability of user U_i. Thus in each slot, user U_i flips a biased coin with probability of heads being p_i and transmits if and only if a heads occurs. Obviously if $p_1 \neq p_2$, the users do not have equal access to the channel.

Let $Q_i(n)$ denote the queue length of user U_i at time slot n. We have the following definition.

Definition 1 *The network is said to be stable [46] under the input rate vector (λ_1', λ_2') if there exists $(p_1, p_2) \in [0,1]^2$ such that*

(a) $\lim_{n\to\infty} \Pr\{Q_i(n) \leq x\} = F_i(x)$

(b) $\lim_{x\to\infty} F_i(x) = 1$.

In other words, the Markov chain described by $Q_i(n)$ has a well-defined stationary distribution.

Suppose that $p_1 = 1$ and $p_2 = 0$. Then U_1 always accesses the channel and U_2 never. In that case, it is easy to see from (3.1) that the total average throughput of U_1 (over all the carriers) in any time slot is C_{max} given by (3.2). To ensure that the queue of user U_1 is stable, by Loynes Theorem [35], it is necessary and sufficient that the input rate is less than the maximum throughput i.e., $\lambda_1' < C_{max}$ or $\frac{\lambda_1'}{C_{max}} < 1$. Henceforth, we define $\lambda_i = \frac{\lambda_i'}{C_{max}}$ to be the normalized input rate for user U_i.

Definition 2 *The stability region $\mathcal{S}_2$ of a two user network is defined as the set of all input rate vectors (λ_1, λ_2) for which the queues are stable, i.e.,*

$$\begin{aligned} \mathcal{S}_2 \;=\; \Big\{ & (\lambda_1, \lambda_2) \in [0,1]^2 : \exists\, (p_1, p_2) \in [0,1]^2 \\ & s.t.\ (a)\ and\ (b)\ in\ Definition\ 1\ are\ satisfied \Big\}. \end{aligned} \tag{3.3}$$

The stability region for the two-user single carrier Aloha network under the collision model has been derived in [4]. Specifically, it has been shown that the stability region $\mathcal{S}_2$ is given by

$$\mathcal{S}_2 = \Big\{ (\lambda_1, \lambda_2) \in [0,1]^2 : \sqrt{\lambda_1} + \sqrt{\lambda_2} < 1 \Big\}. \tag{3.4}$$

Moreover, for each $(\lambda_1, \lambda_2) \in \mathcal{S}_2$, it has also been shown that the corresponding access probabilities of the users, p_1 and p_2, satisfy $p_1 p_2 < 1$. For the same bandwidth, in this chapter, we present a novel scheme using multicarriers to achieve any $(\lambda_1, \lambda_2) \in \mathcal{S}_2$ with $p_1 = p_2 = 1$. This is elegant from the point of view of implementation since the users are free to transmit whenever they have packets to send.

The main advantage of multicarrier networks is that each user U_i has access to more than one carrier. Throughout the chapter, we allow user U_i to select a fraction α_i of the carriers. First, we let the users select the carriers randomly and describe the stability region of such a random access protocol. We then allow the users to select carriers according to the channel gains and show how multicarrier diversity can be exploited to improve upon the stability properties of two user networks.

3.3.1 Random Selection

Let $H_{i,k}$ denote the channel gain of user U_i at the k^{th} carrier. Consider the case when U_1 alone accesses all the carriers. Since U_1 selects a fraction α_1 of the carriers, it is allowed to transmits in $\alpha_1 n$ carriers. When the carriers are chosen randomly, it is easy to see that the maximum (normalized) throughput achievable by user U_1 at the k^{th} carrier is given by

$$\frac{E\{\log_2(1 + H_{1,k})\}}{nC_{max}} = \frac{1}{n}.$$

It follows that the total average throughput of user U_1 is

$$\frac{\sum_{k=1}^{n\alpha_1} E\{\log_2(1 + H_{1,k})\}}{nC_{max}} = \alpha_1.$$

When both U_1 and U_2 are allowed to select carriers independently, there is a possibility that some of the carriers overlap. Let U_1 select $\alpha_1 n$ carriers and U_2 select $\alpha_2 n$ carriers, independently. Further, let p_1 and p_2 be the access probabilities for U_1 and U_2, respectively. Thus, for example, in any time slot, with a probability p_1, user U_1 decides to transmit in $\alpha_1 n$ carriers chosen randomly. Note that each carrier is assumed to be a collision channel. Therefore, in any given time slot, a fraction $\beta \in (0, \min\{\alpha_1, \alpha_2\})$ of the carriers would experience collision and result in zero throughput. Note that β is a random variable since each user is unaware of the other's carriers. We have the following theorem.

Theorem 1 *Consider a two user multicarrier network with users U_1 and U_2 having access probabilities p_1 and p_2, respectively. If U_i selects a fraction α_i of the carriers randomly* *for transmission, the stability region of the resulting protocol is precisely $\mathcal{S}_2$ given by (3.4). Moreover, any $(\lambda_1, \lambda_2) \in \mathcal{S}_2$ can be achieved with $p_1 = p_2 = 1$.*

PROOF: We consider two cases separately.

- $\alpha_1 + \alpha_2 \leq 1$.

  Without loss of generality, let $\alpha_2 < \alpha_1$. Given the channel access probabilities p_1 and p_2 of the two users, it follows that the throughput of user U_1 is given by

  $$p_1(1-p_2)\alpha_1 + p_1 p_2 \int_0^{\alpha_2} (\alpha_1 - \beta) dF_n(\beta). \tag{3.5}$$

  The first term in the above equation is the throughput when U_2 decides not to transmit. The second term in the above equation is the throughput achieved when U_2 also transmits and thus results in a throughput of $(\alpha_1 - \beta)$ for U_1. $F_n(\beta)$ denotes the cumulative distribution function of the random variable β. Each of the $\alpha_1 n$ carriers that is chosen by U_1 has a probability of α_2 of being chosen by U_2. Thus, it can be seen that

  $$F_n(\beta) = \sum_{k \leq \beta n} \begin{pmatrix} \alpha_1 n \\ k \end{pmatrix} \alpha_2^k (1-\alpha_2)^{\alpha_1 n - k}.$$

  It can be shown that as $n \to \infty$, $F_n(\beta) \to \mathbb{1}(\beta > \alpha_1\alpha_2)$, where $\mathbb{1}(.)$ is the indicator function and hence the asymptotic average throughput of U_1 can be obtained from (3.5) as

  $$\eta_1 = p_1(1-p_2)\alpha_1 + p_1 p_2(\alpha_1 - \alpha_1\alpha_2). \tag{3.6}$$

  To determine the stability region of U_1, we refer to a result regarding multipacket reception (MPR) channels from [46]. In Eq. (39) of [46], the average throughput of U_1 is derived to be

  $$\eta_1 = p_1(1-p_2)q_{1|1} + p_1 p_2 q_{1|\{1,2\}}, \tag{3.7}$$

  where $q_{i|i}$ is defined as the probability that U_i is successful given that only U_i transmits and $q_{i|\{1,2\}}$, the probability that U_i is successful given that U_1 and U_2 transmit. A channel is defined to be strong MPR if [46]

  $$\frac{q_{1|\{1,2\}}}{q_{1|1}} + \frac{q_{2|\{1,2\}}}{q_{2|2}} \geq 1. \tag{3.8}$$

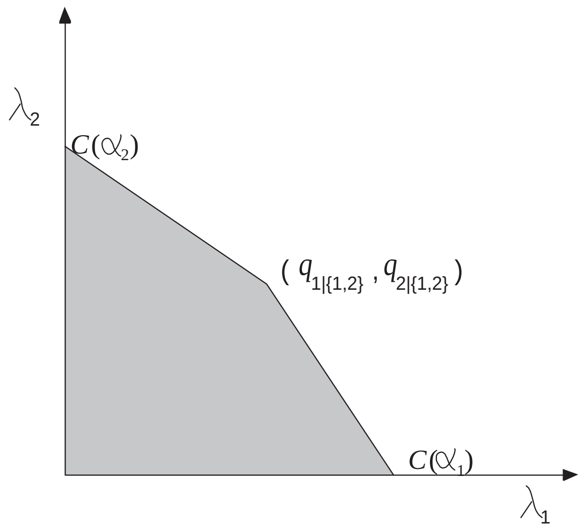

Figure 5: Stability region $\mathcal{T}(\alpha_1, \alpha_2)$ when $\alpha_1 + \alpha_2 \leq 1$

Our case is similar to the situation described above since presence of multiple carriers allow more than one packet to be transmitted in a single slot. The only adjustment we make is that we redefine $q_{i|i}$ to be the average throughput of U_i given that U_i alone transmits and $q_{i|\{1,2\}}$ to be the average throughput achieved by U_i given that U_1 and U_2 transmit. Comparing (3.6) and (3.7), we find that $q_{1|1} = \alpha_1$ and $q_{1|\{1,2\}} = \alpha_1 - \alpha_1\alpha_2$, $q_{2|2} = \alpha_2$ and $q_{2|\{1,2\}} = \alpha_2 - \alpha_1\alpha_2$. Moreover, it can be seen that

$$\frac{q_{1|\{1,2\}}}{q_{1|1}} + \frac{q_{2|\{1,2\}}}{q_{2|2}} = 1 - \alpha_1 + 1 - \alpha_2 \geq 1 \tag{3.9}$$

since we have chosen $\alpha_1 + \alpha_2 \leq 1$ and thus the channel behaves like strong MPR.

It has been shown in Theorem 3 of [46] that if a reception channel behaves like strong MPR, i.e. if (3.8) is satisfied, then the stability region is a convex quadrilateral $\mathcal{T}(\alpha_1, \alpha_2)$ with vertices $(0,0), (q_{1|1}, 0), (q_{1|\{1,2\}}, q_{1|\{1,2\}})$, and $(0, q_{2|2})$ as shown in Figure 5. Moreover $p_1 = p_2 = 1$ achieves any $(\lambda_1, \lambda_2) \in \mathcal{T}(\alpha_1, \alpha_2)$. Therefore, when $\alpha_1 + \alpha_2 \leq 1$, the stability region of our scheme can be found as

$$\mathcal{S}_{r,1} = \bigcup_{0<\alpha_1+\alpha_2\leq 1} \mathcal{T}(\alpha_1, \alpha_2).$$

- $\alpha_1 + \alpha_2 > 1$.

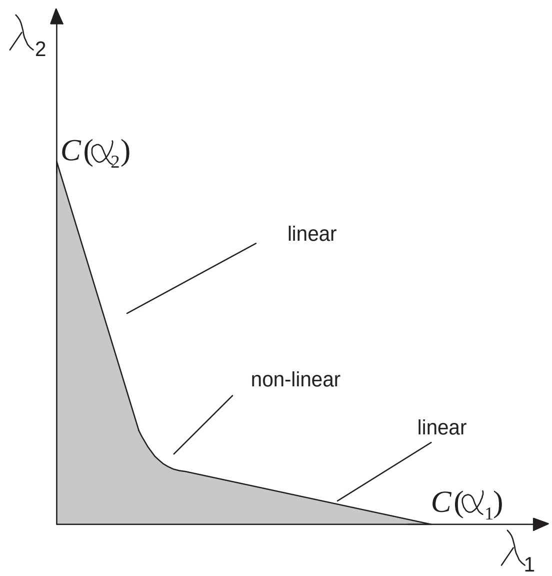

Figure 6: Stability region $\mathcal{T}'(\alpha_1, \alpha_2)$ when $\alpha_1 + \alpha_2 > 1$

When $\alpha_1 + \alpha_2 > 1$, it can be seen from (3.9) that

$$\frac{q_{1|\{1,2\}}}{q_{1|1}} + \frac{q_{2|\{1,2\}}}{q_{2|2}} = 1 - \alpha_1 + 1 - \alpha_2 < 1$$

and the channel is said to be of weak MPR [46]. Given α_1 and α_2 such that $\alpha_1+\alpha_2 > 1$, the stability region $\mathcal{T}'(\alpha_1, \alpha_2)$ is shown in Figure 6. The exact expression for $\mathcal{T}'(\alpha_1, \alpha_2)$ can be found in Lemma 2 of [46] with the appropriate redefinitions already mentioned. The stability region $\mathcal{S}_{r,2}$ of two user multicarrier Aloha with $\alpha_1 + \alpha_2 > 1$ is given by

$$\mathcal{S}_{r,2} = \bigcup_{\alpha_1+\alpha_2>1} \mathcal{T}'(\alpha_1, \alpha_2).$$

From Figure 7, we note that given α_1 and α_2 such that $\alpha_1 + \alpha_2 > 1$,

$$\mathcal{T}'(\alpha_1, \alpha_2) \subset \mathcal{T}(\alpha_1, 1 - \alpha_1) \cup \mathcal{T}(1 - \alpha_2, \alpha_2)$$

where $\mathcal{T}(.,.)$ is as illustrated in Figure 5. Thus it is clear that

$$\mathcal{S}_{r,2} = \bigcup_{\alpha_1+\alpha_2>1} \mathcal{T}'(\alpha_1, \alpha_2) \subset \bigcup_{0<\alpha<1} \mathcal{T}(\alpha, 1 - \alpha) \subset \mathcal{S}_{r,1}.$$

From the above discussion, we deduce that the overall stability region is $\mathcal{S}_r = \mathcal{S}_{r,1} \cup \mathcal{S}_{r,2} = \mathcal{S}_{r,1}$. Further, it can be easily seen that $\mathcal{S}_r = \mathcal{S}_2$, where $\mathcal{S}_2$ is as defined in (3.4). $\square$

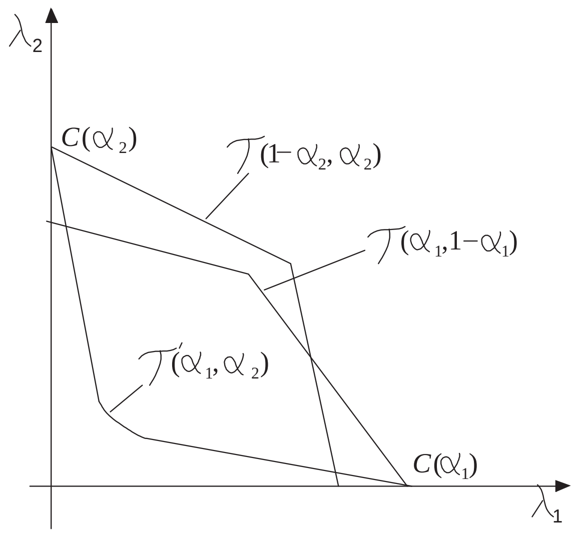

Figure 7: When $\alpha_1 + \alpha_2 > 1$, $\mathcal{T}'(\alpha_1, \alpha_2) \subset \mathcal{T}(\alpha_1, 1-\alpha_1) \cup \mathcal{T}(1-\alpha_2, \alpha_2)$

3.3.2 Ordered Selection

Consider a two user network, U_1 and U_2, with user U_i choosing a fraction α_i of the carriers. In the previous section, we allowed U_i to select the carriers randomly. In this section, we require U_i to select α_i of his *best* carriers. As before, let $H_{i,k}$ denote the channel gain of user U_i at the k^{th} carrier. Arrange the channel gains in decreasing order and call them $G_{i,1}, G_{i,2}, ..., G_{i,n}$. Let U_i select the first $\alpha_i n$ carriers. Define

$$C(\alpha) = \limsup_{n\to\infty} \frac{\sum_{k=1}^{n\alpha} E\{\log_2(1+G_{i,k})\}}{nC_{max}}, \tag{3.10}$$

where C_{max} is given by (3.2). It can be easily seen that when a user selects a fraction α of the best of its carriers, the asymptotic normalized throughput of that user is then given by $C(\alpha)$. In Appendix B, we have proved the following result.

Theorem 2 *$C(\alpha)$ as defined by (3.10) has the following equivalent expression:*

$$C(\alpha) = \frac{\int_{-\ln\alpha}^{\infty} \log_2(1+x)e^{-x}dx}{\int_0^{\infty} \log_2(1+x)e^{-x}dx}. \tag{3.11}$$

Moreover, $C(\alpha)$ satisfies:

(a) $C(0) = 0;\quad C(1) = 1,$

(b) If $\alpha < \alpha'$, $C(\alpha) < C(\alpha')$,

(c) $0 < \alpha < 1$, $C(\alpha) > \alpha$. $\square$

In fact, parts (a) and (b) can be obtained directly from (3.11) and part (c) is proved in Appendix B. Part (c) of the above Theorem ascertains the fact that multicarrier diversity results in increased throughput. This is because if the total number of carriers is n, for large n, $C(\alpha)C_{max}n$ represents the throughput of a particular user that selects a fraction α of its *best* carriers and $\alpha C_{max}n$ represents the throughput when the user selects a fraction α of its carriers *randomly.* In Figure 8, we have plotted the function $C(\alpha)$ as a function of α. It can be seen that $C(\alpha)$ is an increasing function that satisfies $C(\alpha) > \alpha$ for all $0 < \alpha < 1$.

As before, consider now a two user network. When both U_1 and U_2 are allowed to select carriers independently, there is a possibility that some of the carriers overlap. Let U_1 select $\alpha_1 n$ of its best carriers and U_2 select $\alpha_2 n$ of its best carriers, independently. Further, let p_1 and p_2 be the access probabilities for U_1 and U_2, respectively. Again, in any given time slot, a fraction $\beta \in (0, \min\{\alpha_1, \alpha_2\})$ of the carriers would experience collision and result in zero throughput. Also, β is a random variable since each user is unaware of the other's carriers. Following the lines of the proof of Theorem 1, we derive the throughput of user U_1 to be

$$p_1(1-p_2)C(\alpha_1) + p_1 p_2 \int_0^{\alpha_2} C_1(\alpha_1, \beta) dF_n(\beta),$$

where $C_1(\alpha_1, \beta)$ is the throughput of user U_1 given that a fraction β of the carriers experience collision. It is tough to get a closed form expression of $C_1(.,.)$ since the fraction β of carriers experiencing collision could be from anywhere in the fraction α_1. However, $C_1(.,.)$ satisfies the following inequality:

$$C(\alpha_1) - C(\beta) \leq C_1(\alpha_1, \beta) \leq C(\alpha_1 - \beta). \tag{3.12}$$

The lower bound is obtained by assuming that β of the best carriers experience collision and the upper bound is obtained by assuming that β of the worst carriers experience collision. As in the proof of Theorem 1, $F_n(\beta) \to \mathbb{1}(\beta > \alpha_1\alpha_2)$ as $n \to \infty$ and it can be shown that the asymptotic throughput of user U_1 can be bounded above and below as

$$\eta_l \leq \eta_1 \leq \eta_u,$$

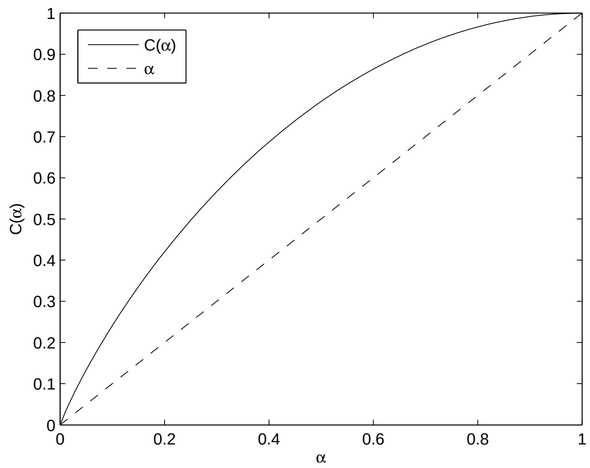

Figure 8: Plot of $C(\alpha)$ vs. α where $C(\alpha)$ is given by (3.11)

where

$$\eta_l = p_1(1-p_2)C(\alpha_1) + p_1p_2(C(\alpha_1) - C(\alpha_1\alpha_2)), \tag{3.13}$$

and

$$\eta_u = p_1(1-p_2)C(\alpha_1) + p_1p_2C(\alpha_1 - \alpha_1\alpha_2). \tag{3.14}$$

In what follows, we shall derive the upper and lower bounds on the stability region of two user multicarrier networks following ordered selection.

3.3.3 Lower Bound

As in proof of Theorem 1, we define $q_{i|i}$ to be the throughput of user U_i given that U_i alone transmits and $q_{i|\{1,2\}}$ to be the throughput of user U_i given that both U_1 and U_2 transmit. Comparing (3.7) and (3.13) it can be seen that $q_{1|1} = C(\alpha_1)$ and $q_{1|\{1,2\}} = C(\alpha_1) - C(\alpha_1\alpha_2)$. Similarly, $q_{2|2} = C(\alpha_2)$ and $q_{2|\{1,2\}} = C(\alpha_2) - C(\alpha_1\alpha_2)$. From (3.8), we find that the multicarrier network behaves like strong MPR [46] if

$$\frac{C(\alpha_1) - C(\alpha_1\alpha_2)}{C(\alpha_1)} + \frac{C(\alpha_2) - C(\alpha_1\alpha_2)}{C(\alpha_2)} \geq 1.$$

Simplifying the above equation, we define

$$\mathcal{I}_l = \left\{ (\alpha_1, \alpha_2) \in [0,1]^2 : \frac{1}{C(\alpha_1)} + \frac{1}{C(\alpha_2)} \leq \frac{1}{C(\alpha_1\alpha_2)} \right\}$$

to be the set of all (α_1, α_2) for which the channel behaves like strong MPR. Recall that in the random selection, $\alpha_1 + \alpha_2 \leq 1$ defines the set for which the channel behaves like strong MPR. For each $(\alpha_1, \alpha_2) \in \mathcal{I}_l$, it follows from Theorem 3 of [46] that the stability region is the convex quadrilateral $\mathcal{T}(\alpha_1, \alpha_2)$ as in Figure 5. Define

$$\mathcal{S}_{o,l} = \bigcup_{(\alpha_1,\alpha_2)\in\mathcal{I}_l} \mathcal{T}(\alpha_1, \alpha_2)$$

to be the lower bound for the stability region of two user multicarrier Aloha network following ordered selection.

It is important to see that if $(\alpha_1, \alpha_2) \in [0,1]^2 - \mathcal{I}_l$, i.e., if the channel behaves like weak MPR, the stability region $\mathcal{T}'(\alpha_1, \alpha_2)$ is as shown in Figure 6. By arguments similar to the proof in Theorem 1, it can be shown that

$$\bigcup_{(\alpha_1,\alpha_2)\in[0,1]^2-\mathcal{I}_l} \mathcal{T}'(\alpha_1, \alpha_2) \subset \mathcal{S}_{o,l}. \tag{3.15}$$

Intuitively the above result suggests that allowing both U_1 and U_2 to choose a large fraction of carriers does not improve the stability region. We are now in a position to state the major result of this chapter.

Theorem 3 *The stability region of two user multicarrier Aloha network with ordered selection strictly contains the stability region of two user multicarrier Aloha network with random selection.*

PROOF: To prove the theorem, it suffices to prove that

$$\overline{\mathcal{S}}_2^c \bigcap \mathcal{S}_{o,l} \neq \phi, \tag{3.16}$$

where $\mathcal{S}_2$ is defined in (3.4). To that end, we prove that the boundary of $\mathcal{S}_2$, is contained in $\mathcal{S}_{o,l}$. In fact, if $(\lambda_1, \lambda_2) \in \overline{\mathcal{S}}_2 - \mathcal{S}_2$, it is easy to see from (3.4) that there exists $\gamma \in [0,1]$ such that $\lambda_1 = \gamma^2$ and $\lambda_2 = (1-\gamma)^2$. If we can prove that there exists $(\alpha_1, \alpha_2) \in \mathcal{I}_l$ such that $(\gamma^2, (1-\gamma)^2) \in \mathcal{T}(\alpha_1, \alpha_2)$, then we are done. We have proved the following claim in Appendix B.

Claim: For any $\gamma \in [0,1]$, there exists $(\alpha_1, \alpha_2) \in [0,1]^2$ such that $C(\alpha_1) - C(\alpha_1\alpha_2) = \gamma^2$,

$C(\alpha_2) - C(\alpha_1\alpha_2) = (1-\gamma)^2$, and $\frac{1}{C(\alpha_1)} + \frac{1}{C(\alpha_2)} = \frac{1}{C(\alpha_1\alpha_2)}$.

It is easy to see that if the above claim holds, then $(\gamma^2, (1-\gamma)^2) \in \mathcal{S}_{o,l}$ and hence every point in the boundary of $\mathcal{S}_2$ is contained in $\mathcal{S}_{o,l}$ which proves that $\overline{\mathcal{S}}_2 \subseteq \mathcal{S}_{o,l}$.

To prove that the containment is strict, we let $\alpha_1 = \alpha_0$ where α_0 is the solution to $C'(\alpha_0) = 1$. Also, we let $\alpha_2 = \alpha_0'$ where

$$\frac{1}{C(\alpha_0)} + \frac{1}{C(\alpha_0')} = \frac{1}{C(\alpha_0\alpha_0')}. \tag{3.17}$$

From Figure 5, we find that the stability region $\mathcal{T}(\alpha_0, \alpha_0')$ is given by the straight line,

$$\frac{\lambda_1}{C(\alpha_0)} + \frac{\lambda_2}{C(\alpha_0')} = 1.$$

Obviously $(\lambda_1^0, \lambda_2^0) = \left(\frac{C(\alpha_0)C(\alpha_0\alpha_0')}{C(\alpha_0')}, \frac{C(\alpha_0')C(\alpha_0\alpha_0')}{C(\alpha_0)}\right) \in \mathcal{T}(\alpha_0, \alpha_0')$ and we see from (3.17) that

$$\sqrt{\lambda_1^0} + \sqrt{\lambda_2^0} = C(\alpha_0) + C(\alpha_0') = \frac{C(\alpha_0)C(\alpha_0')}{C(\alpha_0\alpha_0')}.$$

If we can show that $\frac{C(\alpha_0)C(\alpha_0')}{C(\alpha_0\alpha_0')} > 1$, then $(\overline{\mathcal{S}}_2 - \mathcal{S}_2)^c \cap \mathcal{T}(\alpha_0, \alpha_0') \neq \phi$ and this completes the proof. Consider the function $f(\alpha) = C(\alpha)C(\alpha_0) - C(\alpha\alpha_0)$ for $\alpha \in [0,1]$. It can be seen that

$$\begin{aligned} f''(\alpha) &= \frac{1}{C_{max}\log_e 2}\frac{1}{\alpha}\left(\frac{\alpha_0}{1-\ln\alpha\alpha_0} - \frac{C(\alpha_0)}{1-\ln\alpha}\right). \\ &< \frac{1}{C_{max}\log_e 2}\frac{\alpha_0 - C(\alpha_0)}{\alpha(1-\ln\alpha)} < 0 \end{aligned}$$

from Part (c) of Theorem 2. Thus $f(.)$ is a non-zero concave function on $(0,1)$ with $f(0) = f(1) = 0$. It follows that $f(\alpha_0') > 0$ or $C(\alpha_0)C(\alpha_0') > C(\alpha_0\alpha_0')$. $\square$

3.3.4 Upper Bound

To compute the upper bound, we proceed in a similar manner as the lower bound. Comparing (3.7) and (3.14) it can be seen that $q_{1|1} = C(\alpha_1)$ and $q_{1|\{1,2\}} = C(\alpha_1 - \alpha_1\alpha_2)$. Similarly, $q_{2|2} = C(\alpha_2)$ and $q_{2|\{1,2\}} = C(\alpha_2 - \alpha_1\alpha_2)$. We define

$$\mathcal{I}_u = \left\{ (\alpha_1, \alpha_2) \in [0,1]^2 : \frac{C(\alpha_1 - \alpha_1\alpha_2)}{C(\alpha_1)} + \frac{C(\alpha_2 - \alpha_1\alpha_2)}{C(\alpha_2)} \geq 1 \right\}$$

to be the set of all (α_1, α_2) for which the channel behaves like strong MPR. For each $(\alpha_1, \alpha_2) \in \mathcal{I}_u$, it follows from Theorem 3 of [46] that the stability region is the convex

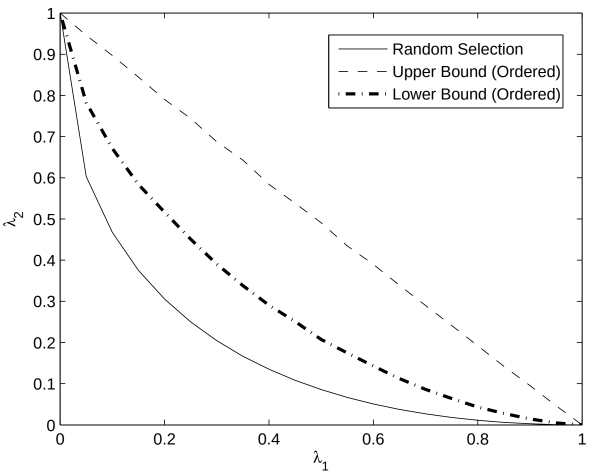

Figure 9: Stability regions for random and ordered selection

quadrilateral $\mathcal{T}(\alpha_1, \alpha_2)$ as in Figure 5. Define

$$\mathcal{S}_{o,u} = \bigcup_{(\alpha_1,\alpha_2)\in\mathcal{I}_u} \mathcal{T}(\alpha_1, \alpha_2)$$

to be the lower bound for the stability region of two user multicarrier Aloha network following ordered selection. By arguments similar to the proof in Theorem 1, it can be shown that

$$\bigcup_{(\alpha_1,\alpha_2)\in[0,1]^2-\mathcal{I}_u} \mathcal{T}'(\alpha_1, \alpha_2) \subset \mathcal{S}_{o,u}, \tag{3.18}$$

where $\mathcal{T}'(\alpha_1, \alpha_2)$ as shown in Figure 6 is the stability region when the channel behaves like weak MPR.

In Figure 9, we plot the upper and lower bounds for the stability region of the two user multicarrier Aloha network with ordered selection.

3.4 Conclusion

In this chapter, by allowing for multiple carriers, we have provided a simple 1-persistent scheme to achieve any input rate vector in the stability region of two user Aloha network. This can be achieved by allowing each user to select a fraction of the carriers randomly and transmit whenever there is a packet to send. We have then studied the effect of multicarrier diversity and shown that ordered selection based on channel gains can lead to increase in the stability region of two user Aloha networks. Specifically, we have provided upper and lower bounds on the achievable rate region with ordered selection.

CHAPTER IV

RANDOM ACCESS IN CAPTURE NETWORKS

4.1 Introduction

In [14] and the previous chapters, we have shown the power of the CAMCRA scheme for a collision channel model. In the collision channel model [7], if more than one packet is transmitted simultaneously, all the packets are assumed to be lost. The collision channel model allows for tractable analysis and lends insight into the behaviour of the random access scheme. In many physical channels, however, it is possible to recover at least one packet even in the presence of multiple interferers. This is commonly referred to as the capture effect [23, 29, 40]. For example, in the presence of fading, the varying power levels of received packets can be exploited to increase the packet reception probability [23, 29, 22]. For single antenna networks, at most one packet can be resolved in the event of interference [30, 2, 11]. In this chapter, we illustrate the power of the CAMCRA scheme for stabilizing single antenna networks with capture.

To analyze stability of networks with capture, two popular models have been used in the literature: *infinite user model* and *finite user model*. For the infinite user model, the total number of packets in the network awaiting transmission (backlogged packets) is modelled as a Poisson random variable with finite mean. To stabilize the network, the packet transmission probability is adjusted dynamically to avoid unbounded increase in the number of backlogged packets. These networks have been discussed in [29, 40, 3] and the references therein.

For the finite user model, the total number of users is fixed and each user is assumed to be equipped with a buffer capable of holding arbitrarily large but fixed number of packets [50, 49, 46]. The goal is then to design a random access scheme so that buffer of each user does not overflow. From the results reported in [2], the finite user random access networks with capture is unstable for most fading channels in practice if the transmission probability is independent of the network population. While it is implicit in the work of [2], it has been explicitly stated and proved in [38] for Rayleigh faded networks. In [38], a population dependent transmission control has been provided as an alternative to stabilize random access networks with capture. However, the population sometimes varies dynamically with time and its information is not available. The population dependent transmission control in [38] is not feasible in this case. Furthermore, the capture channel considered in [38] is capable of *multipacket reception* (MPR), i.e., more than one packet can be resolved in the event of collision. Most MPR channels require advanced signal processing techniques that may be costly to implement on a large scale in practice. Therefore, in this chapter we show how the CAMCRA scheme can be used as a population *independent* transmission control scheme to stabilize random access networks with capture capable of at most single packet reception [13].

The chapter is organized as follows: In Section 4.2, we provide sufficient motivation for the problem considered in this chapter. We also describe the capture effect in fading channels and define the packet reception probability. In Section 4.3, we study the stability properties of multicarrier networks with capture under the NS-RA scheme described in Section 2.4 of Chapter II. In particular, we show that the network experiences instability under existing transmission control schemes without knowledge of network population. In Section 4.4, we introduce a generic random access scheme [14] suited for multicarrier networks. The scheme essentially allows for channel selection based on instantaneous gains. For ease of analysis, we define a parameter called the population density and consider networks with different population densities: optimally populated, densely populated, and sparsely populated. In Section 4.4.1, we prove that the proposed population independent transmission control scheme stabilizes random access networks with capture by deriving a concise expression for the maximum achievable throughput. We discuss the effect of multicarrier diversity on stability region of the proposed scheme in Section 4.4.6. Finally, we conclude the chapter in Section 4.5.

4.2 Problem Motivation

Consider a random access network consisting of n users occupying a bandwidth of B Hz wanting to access a base station (BS). In this chapter, we assume that all users experience Rayleigh fading and that the transmission is slotted into intervals of equal length. Moreover, channel fades corresponding to different users are assumed to be independent as in [18]. If a signal x is sent, the received signal y is given by

$$y = \sqrt{h}e^{j\theta}x + w,$$

where h denotes the channel power, θ is a random phase term, and w denotes the additive noise. In this chapter, w is assumed to be white Gaussian with zero mean and variance, $E\{|w|^2\} = N_0$. We also assume that each user has access to its channel gain, h. This is facilitated by allowing pilot symbols to be transmitted at regular intervals [18].

Let $\{h_k\}_{k=1}^n$ denote the instantaneous channel gains of the users contending for a particular channel in a particular time slot. In this chapter, we assume that h_k are independent and identically distributed (i.i.d.) with *finite* mean i.e., $E\{h_k\} < \infty$. Let $\{s_k\}_{k=1}^n$ denote the messages (symbols) of the n users with $E\{s_k\} = 0$ and $E\{|s_k|^2\} = P_0$. For simplicity we let $P_0 = 0$ dB and observe that the received signal y is given by

$$y = \sum_{k=1}^{n} \sqrt{h_k}e^{j\theta_k}s_k + w \tag{4.1}$$

where w is the additive white noise. Various definitions of successful packet capture [23, 39, 29, 22, 2, 9, 6, 38, 55] have appeared in the literature over the years (see [48] for a detailed description). We follow the definition in [22].

Definition: Consider a random access network with n users contending for a particular carrier. For the received signal y given by (4.1), let $P_k = P_k(h_1, h_2, ..., h_k)$ denote the symbol error probability of the k^{th} user. Given $\epsilon > 0$, the symbol (packet) of the k^{th} user is successfully captured if and only if

$$P_k < \epsilon. \tag{4.2}$$

In [23, 29, 2], packet of user k is said to be successfully captured if the received signal to interference and noise ratio (SINR) is greater than a certain pre-determined threshold, i.e.,

$$\frac{h_k}{N_0 + \sum_{k \neq i} h_i} > \lambda \tag{4.3}$$

for some $\lambda > 0$. When the symbols s_k in (4.1) are chosen from a fixed constellation, we show that (4.3) and (4.2) are equivalent provided the multiuser interference is Gaussian. In fact, for a particular channel realization $h_1, h_2, ..., h_k$ in (4.1), the probability of error for user k is given by

$$P_k = Q\left(\sqrt{\gamma \frac{h_k}{N_0 + \sum_{k \neq i} h_i}}\right),$$

where the constant γ depends on the constellation size and $Q(.)$ is the error function which is strictly decreasing.We find that (4.3) and (4.2) are equivalent with $\lambda = \frac{(Q^{-1}(\epsilon))^2}{\gamma}$. For any reasonable target error probability ϵ in practical communication networks with a single antenna at the receiver, it can be ensured that $\lambda > 1$ or, equivalently, at most one packet is captured by the channel [30, 2]. The definition given by (4.2) is more general and encompasses multicarrier networks as will be seen later.

4.2.1 Stability of finite user random access with capture

In the finite user model [7], the total number of users in the network n is fixed. In any time slot, each user transmits with a probability p_n. Let a_n denote the average number of packets successfully received when n users contend for a particular channel. Obviously, in single packet reception channels $a_n < 1$ and is interpreted as the packet reception probability. The quantity $a = \lim_{n\to\infty} a_n$ denotes the maximum achievable throughput [40, 7, 38] of the network assuming a finite user model. By Loynes Theorem [35], the maximum achievable throughput is in turn equal to the maximum allowable input rate of the random access network.

The stability issue for random access networks with capture for finite user model has been discussed in [2, 38]. In [2], packet reception models are considered where the probability distribution of the channel gain h_k of user k has a non-zero roll-off factor δ, i.e.,

$$\lim_{h_0 \to \infty} \Pr\{h_k > h_0\} h_0^{\delta} = c \tag{4.4}$$

for some constant $c < \infty$. Allowing a packet transmission probability of $p_n = 1$, it is shown (see Proposition 3.1 of [2]) that $a = \lim_{n\to\infty} a_n > 0$ if and only if $0 < \delta < 1$. Thus random access networks with capture are stable if and only if the roll-off factor for the channel distribution is less than unity. However, as pointed out in [55], it is easy to see that if the roll-off factor satisfies $0 < \delta < 1$ then the average channel gain of any user is infinite, i.e., $E\{h_k\} = \infty$! In any physically meaningful model it is reasonable to assume that the average channel gain of any user is finite[1]. The results stated in [2] still hold for any transmission probability $p_n = p < 1$ (independent of n) and any single packet reception channel. In [38], multipacket reception models are considered for random access networks with capture. In particular, *code division multiple access* (CDMA) networks with *linear minimum mean-square error* (LMMSE) receivers are considered and population *dependent* (p_n not a constant) transmission scheme is proposed as an alternative to stabilize random access networks with capture. Usually, channels capable of resolving multiple packets require advanced signal processing techniques that significantly increase the network complexity and cost. Also, in many practical channels, the network population may vary dynamically and it may not be possible to obtain a very accurate estimate of the network size.

This motivates the need for stabilizing finite user random access networks operating on channels capable of resolving at most one packet per slot. Also, it is desired that the scheme be population independent. In this chapter, we show that the CAMCRA scheme [14] is an effective population *independent* transmission control scheme that stabilizes finite user single packet reception random access networks with capture.

4.3 Multicarrier Networks with NS-RA

Consider a network with N_u users and N_c carriers occupying a total bandwidth B Hz. In the NS-RA scheme described in Section 4.4, each user transmits on a carrier of bandwidth $\frac{B}{N_c}$ Hz and attains a rate of $\frac{R}{N_c}$ bits/sec on any particular carrier, where R denotes the overall transmission rate. Also, it is obvious that in each of the N_c carriers, all the N_u

[1] In much of the literature, e.g. [38, 2, 23], for a user k at a distance r_k from the BS, the channel gain h_k is modelled as $h_k = he^{\xi}Kr_k^{-\beta}$, where h is exponentially distributed random variable, ξ is a Gaussian random variable with zero mean and finite variance, K denotes the path loss constant, and $\beta > 2$ is the path loss exponent. In fact, h models the Rayleigh fading component, e^{ξ} accounts for shadowing, and $Kr_k^{-\beta}$ is the signal attenuation due to path loss. To compute the asymptotic throughput $a = \lim_{n\to\infty} a_n$, usually, πr_k^2 is assumed to be uniformly distributed in $[\pi r_0^2, \pi r_1^2]$ for some $r_1 > r_0 \geq 0$ [22, 23, 2]. In fact, Proposition 3.1 of [2] implies that $a > 0$ if and only if $r_0 = 0$. However, $r_0 = 0$ and $\beta > 2$ again implies $E\{h_k\} = \infty$!

users contend for packet transmission. For a particular carrier j, let $h_1, h_2, ..., h_{N_u}$ denote the instantaneous channel gains of the users. It can be easily seen that when the channel is able to capture at most one packet, the power level of the successfully captured packet is the largest [11] and hence from (4.3) we find that

$$\frac{h_{\max}}{N_0 + \sum_k h_k - h_{\max}} > \lambda.$$

and the probability of capture is given by

$$a_{N_u} = \Pr\left\{\frac{h_{\max}}{N_0 + \sum_k h_k - h_{\max}} > \lambda\right\}, \tag{4.5}$$

where $h_{\max} = \max_{1 \le k \le N_u} h_k$. Since each packet is transmitted at a rate $\frac{R}{N_c}$, the throughput per carrier is $\frac{a_{N_u} R}{N_c}$ and the overall network throughput is $N_c \frac{a_{N_u} R}{N_c} = a_{N_u} R$. For simplicity, henceforth we assume that $R = 1$ and study the behaviour of the network throughput a_{N_u} as N_u varies. If all the users in the network undergo independent and identical fading, then we can prove the following proposition regarding a_{N_u} (see Appendix C).

Proposition 7 *Consider a network with N_u users and N_c carriers. In the NS-RA scheme, all the N_u users contend in each of the N_c carriers and the packet reception probability in any carrier is a_{N_u} given by (4.5). We have that as $N_u \to \infty$,*

$$a_{N_u} \to 0. \quad \square$$

As mentioned before, the quantity $\lim_{N_u \to \infty} a_{N_u}$ denotes the maximum allowable input rate of the random access network. Thus the slotted random access networks with capture following the NS-RA scheme is unstable for any non-zero input rate. In what follows, we show that the CAMCRA scheme does not suffer from such instability properties.

4.4 Multicarrier Networks with CAMCRA

The main idea of this chapter is to illustrate the improved stability properties of random access networks employing the CAMCRA scheme (see Section 2.3 of Chapter II).

In the CAMCRA scheme, we recall that each user is allowed to select the carrier with the best instantaneous gain. This can be generalized to allow each user to select $c \geq 1$

carriers. In the CAMCRA scheme with *random selection*, we allow the users to select the c carriers *randomly*. In the CAMCRA scheme with *ordered selection*, each user orders his channel gains in descending order and chooses the first c carriers. First, we discuss the CAMCRA scheme with random selection and show improvement in stability region over the NS-RA scheme. Then, we discuss the CAMCRA scheme with ordered selection.

4.4.1 Random Selection

In the previous section, we have seen that the existing transmission control schemes introduce instability in random access networks with capture. In this section, we prove that the CAMCRA scheme proposed in this chapter stabilizes such networks with capture and has a transmission control that is population independent. Here, we consider the CAMCRA scheme with random selection. In Section 4.4.6, we consider the CAMCRA scheme with ordered selection.

For the rest of this section, we let the carriers of each user to be i.i.d. with finite average channel gain. Since the carriers for user 1 are i.i.d., then the probability that a particular carrier j is chosen for transmission is given by $p = \frac{1}{N_c}$ where N_c refers to the total number of carriers. Let n_j denote the number of users contending for the j^{th} carrier. Since each of the N_u users has a probability p of choosing carrier j, it can be easily seen that

$$\Pr\{n_j = k\} = \begin{pmatrix} N_u \\ k \end{pmatrix} \left(\frac{1}{N_c}\right)^k \left(1 - \frac{1}{N_c}\right)^{N_u - k} \tag{4.6}$$

and that the average number of packets received successfully per carrier is

$$T_{N_u} = \sum_{k=1}^{N_u} a_k \Pr\{n_j = k\} \tag{4.7}$$

where a_k is given by (4.5).

For the ease of analysis, we define the population density of a network with N_u users and N_c carriers as

$$\alpha = \frac{N_u}{N_c}.$$

In this chapter, we consider three possible ranges of population density [14]:

(a) Optimally populated: $1 \leq \alpha < \infty$.

(b) Densely populated: $\alpha \to \infty$.

(c) Sparsely populated: $\alpha < 1$.

4.4.2 Optimally Populated Networks

In optimally populated networks, the total network population is of the same order of the number of carriers, i.e., $\frac{N_u}{N_c} = \alpha$ is fixed. To study the stability properties of such networks, it is necessary to obtain the asymptotic behaviour of the average packet reception probability, T_{N_u}, given by (4.7). Regarding the asymptotic capture probability of multicarrier networks following the CAMCRA scheme with random selection, we can prove the following important result (see Appendix C).

Proposition 8 *Consider an optimally populated slotted random access network with capture consisting of N_u users and N_c carriers. Let T_{N_u} given by (4.7) denote the average number of successfully received packets per carrier. Then for any $\alpha > 0$, as $N_u, N_c \to \infty$, we have*

$$T_{N_u} \to \zeta(\alpha) \tag{4.8}$$

where

$$\zeta(\alpha) = e^{-\alpha} \sum_{k=1}^{\infty} \frac{\alpha^k}{k!} a_k, \tag{4.9}$$

and a_k is given by (4.5). $\square$

Even though $a_n \to 0$ as $n \to \infty$, we find that $\zeta(\alpha) \geq e^{-\alpha}\alpha > 0$ for any finite value of the network density α. Note that for a particular network density α, $\zeta(\alpha) > 0$ represents the maximum allowable input rate that does not result in buffer overflow. In fact, if the random access network consists of N_u users and N_c carriers, N_u, N_c large, the total input rate of the network can be as high as $\zeta\left(\frac{N_u}{N_c}\right)$ without causing buffer overflow for any user. Moreover, any user that has a packet to transmit does so at the beginning of each time slot with probability one irrespective of the number of users contending. Thus we have stabilized random access networks with capture using a population independent transmission control scheme.

So far, we have considered optimally populated networks where the population density α is finite. In what follows, we let $\alpha \to \infty$ (densely populated) and $\alpha \to 0$ (sparsely populated) and study the stability properties on the CAMCRA scheme with random selection. It is important to note from the description in Section 4.4 that the NS-RA scheme has the same maximum achievable throughput (of zero) irrespective of the population density α.

4.4.3 Densely Populated Networks

When the network is densely populated, the number of users occupying the network N_u far exceeds the available number of carriers N_c, i.e., $N_c = o(N_u)$. From (4.7), we find that for any $N_u \geq N_c$, the average number of packets successfully received is

$$T_{N_u} = \sum_{k=1}^{N_u} a_k \begin{pmatrix} N_u \\ k \end{pmatrix} \left(\frac{1}{N_c}\right)^k \left(1 - \frac{1}{N_c}\right)^{N_u - k}.$$

In Appendix C, we have proved the following proposition regarding the throughput of densely populated networks.

Proposition 9 *Consider a densely populated random access network with capture with N_u users and N_c carriers. Let T_{N_u} given by (4.7) denote the average number of successfully received packets per carrier. Then as $N_u, N_c \to \infty$ with $N_c = o(N_u)$, we have*

$$T_{N_u} \to 0. \quad \square$$

Thus we find that densely populated random access networks with capture following the CAMCRA scheme with random selection perform as poorly as the NS-RA scheme. This is intuitive since at large population densities, the average number of users contending for any single channel in the CAMCRA scheme is comparable to that of the NS-RA scheme.

4.4.4 Sparsely Populated Networks

By sparsely populated networks, we mean that the number of carriers available is more than the total number of users in contention, i.e., $N_c \geq N_u$. When the network is sparsely populated, we allow each user to choose more than one carrier. Let each user choose $c > 1$ carriers randomly to transmit packets. It can be easily seen that the probability a particular carrier j is chosen by user 1 is

$$p = \frac{c}{N_c}.$$

Let n_j denote the number of users contending for a particular carrier j. It can be easily seen that

$$\Pr\{n_j = k\} = \begin{pmatrix} N_u \\ k \end{pmatrix} \left(\frac{c}{N_c}\right)^k \left(1 - \frac{c}{N_c}\right)^{N_u - k}.$$

Hence the average number of packets successfully received per carrier in a sparsely populated network following the CAMCRA scheme with random selection is given by

$$\begin{aligned} T'_{N_u} &= \sum_{k=1}^{N_u} a_k \Pr\{n_j = k\} \\ &= \sum_{k=1}^{N_u} a_k \begin{pmatrix} N_u \\ k \end{pmatrix} \left(\frac{c}{N_c}\right)^k \left(1 - \frac{c}{N_c}\right)^{N_u - k} \end{aligned} \tag{4.10}$$

where a_k is given by (4.5). In fact, if for some $\alpha > 0$, each user chooses $c = \frac{N_c}{N_u}\alpha$ carriers, we find that $N_u \frac{c}{N_c} = \alpha$ and the network behaves like an optimally populated network with density α. Therefore, from Proposition 8, we can determine the maximum achievable throughput when the network is sparsely populated.

Proposition 10 *Consider a sparsely populated random access network with capture with N_u users and N_c carriers. Let each user select $c = \frac{N_c}{N_u}\alpha$ carriers to transmit packets. Let T'_{N_u} given by (4.10) denote the average number of packets successfully received per carrier. Then as $N_u, N_c \to \infty$, we have*

$$T'_{N_u} \to \zeta(\alpha),$$

where $\zeta(.)$ is given by (4.9). □

We now determine the optimal value of α that maximizes the throughput $\zeta(\alpha)$. In fact, in Appendix C, we have proved the following proposition regarding the maximal value.

Proposition 11 *Let a_n be any sequence with the following properties:*

(a) $\lim_{n\to\infty} a_n = a \geq 0$,

(b) $a_n \geq a$, $\forall\, n$ *and*

(c) $\exists\, n_0$ *s.t.* $a_{n_0} > a$.

Define ζ^* *as*

$$\zeta^* = \sup_{\alpha > 0} \zeta(\alpha) \tag{4.11}$$

where $\zeta(\alpha) = e^{-\alpha} \sum_{n=1}^{\infty} \frac{\alpha^n a_n}{n!}$.

Then we have,

(a) $\zeta^* > a$

(b) $\exists\, \alpha^* < \infty$ *s.t.* $\zeta(\alpha^*) = \zeta^*$

(c) $\exists\, \alpha_0 < \infty$ *s.t.* $\forall\, \alpha > \alpha_0$, *we have* $\zeta(\alpha) > 1$. $\square$

Note that for the random access network with capture, the sequence a_n given by (4.5) satisfies the requirements of the above proposition. The term ζ^* defined by (4.11) is the maximum throughput achievable from the CAMCRA scheme. The above theorem states the the maximum is achievable at a finite population density α^*. Therefore, if each user is allowed to select $c = \frac{N_c}{N_u}\alpha^*$ carriers, we find that the sparsely populated network achieves an overall throughput of $\zeta^* = \zeta(\alpha^*)$.

We also see that beyond a certain value of the population density the CAMCRA scheme is always beneficial, though however, the effects are reduced at very high population density.

4.4.5 Example: Rayleigh Fading

For illustration, let us consider a typical Rayleigh fading scenario in which N_u users contend for a particular carrier. For purposes of clarity, we shall consider the case where the channel gains of each user are i.i.d. exponentially distributed with unit mean. If $h_1, ..., h_{N_u}$ denote the i.i.d. instantaneous channel gains for the users, then $\Pr\{h_1 > h\} = e^{-h}$. Also, if $h_{\max} = \max_{1 \le k \le N_u} h_k$, then $\Pr\{h_{\max} > h\} = 1 - (1 - e^{-h})^{N_u}$. Let $S_{N_u} = \sum_{k=1}^{N_u} h_k$. If a_{N_u}

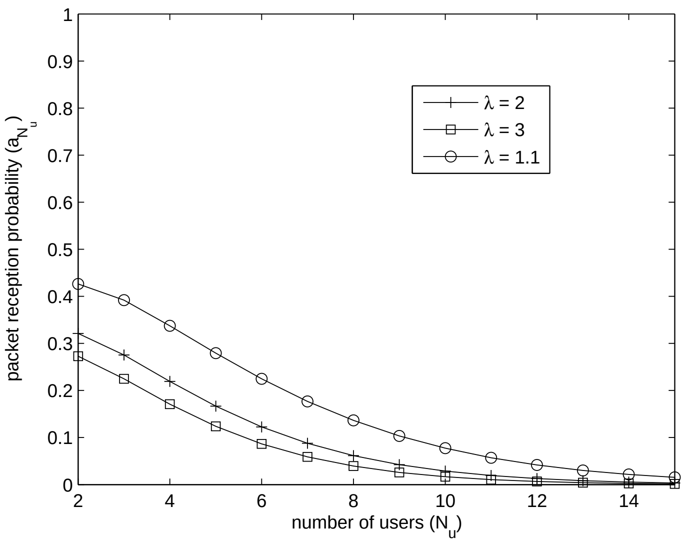

Figure 10: Throughput of random access networks with capture following the NS-RA scheme for various values of threshold λ.

given by (4.5) denotes the packet reception probability, then for any $N_0, \lambda > 0$,

$$\begin{aligned} a_{N_u}^{ray} &= \Pr\left\{\frac{h_{\max}}{N_0 + \sum_{k\neq \max} h_k} > \lambda\right\} = \Pr\left\{h_{\max} > \frac{\lambda N_0 + \lambda S_{N_u}}{1+\lambda}\right\} \\ &= \int_0^{\infty} \left\{1 - (1 - e^{\frac{-\lambda N_0 - \lambda t}{1+\lambda}})^{N_u}\right\} f_{S_{N_u}}(t)dt \\ &= \sum_{k=1}^{N_u} \left(\begin{array}{c} N_u \\ k \end{array}\right) (-1)^{k+1} e^{\frac{-\lambda k N_0}{1+\lambda}} \left(\frac{\lambda+1}{\lambda+1+\lambda k}\right)^{N_u}. \end{aligned} \quad (4.12)$$

It can be easily shown that $a_{N_u}^{ray}$ is bounded above by the first term in (4.12) and hence we have

$$a_{N_u}^{ray} < N_u e^{\frac{-\lambda N_0}{1+\lambda}} \left(\frac{1+\lambda}{1+2\lambda}\right)^{N_u} \to 0$$

as $N_u \to \infty$. In Figure 10, assuming a value of $N_0 = 0$ dB, we have plotted the packet reception probability for Rayleigh faded networks with capture as a function of the network size N_u for various values of threshold λ, which depends on system parameters.

In the NS-RA scheme with N_u users and N_c carriers, we have already seen that the average packet reception probability in any carrier is precisely $a_{N_u}^{ray}$. Since the average packet reception probability is asymptotically equal to the maximum allowable input rate [7], from the above discussion it is apparent that the maximum allowable input rate in the

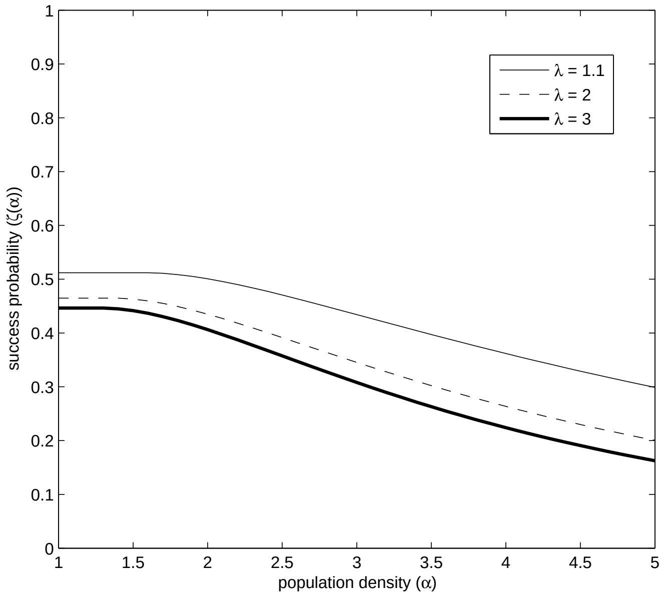

Figure 11: Throughput of random access networks with capture following the CAMCRA scheme with random selection for various values of threshold λ.

NS-RA scheme is zero. However, in the CAMCRA scheme with random selection, for a population density α, it follows from Proposition 2 that the maximum allowable input rate is $\zeta^{ray}(\alpha) = e^{-\alpha} \sum_{k=1}^{\infty} \frac{\alpha^k a_k^{ray}}{k!}$. In Figure 11, we have plotted $\zeta^{ray}(\alpha)$ as a function of α assuming that $N_0 = 0$ dB for different values of the threshold λ. It can be inferred from the figure that as long the population density is less than a certain critical value, it is possible to attain the maximum throughput. This is achieved by allocating more channels to each user as discussed in sparsely populated networks.

4.4.6 Ordered Selection

Consider a network with N_u users and N_c carriers with $N_u \geq N_c$. In the CAMCRA scheme with ordered selection, each user orders the channel gains in descending order and chooses a carrier. The carrier chosen has the best channel gain among all other carriers for that user.

Suppose that $h_1, ..., h_n$ denotes the instantaneous channel gain for user i on carrier j.

Then, as in (4.5), the packet reception probability is given by

$$a_n^{ord} = \Pr\left\{\frac{h_{\max}}{N_0 + \sum_k h_k - h_{\max}} > \lambda\right\} \tag{4.13}$$

where $h_{\max} = \max_{1 \le k \le n} h_k$. For a fixed n, in Appendix C, we have proved the following interesting fact regarding the behaviour of a_n^{ord} as $N_c \to \infty$.

Proposition 12 *Consider a network with N_u users and N_c carriers following the CAMCRA scheme with ordered selection. Given that n users contend for a particular carrier, the average number of successfully received packets is a_n^{ord} given by (4.13). For a fixed n, as $N_c \to \infty$, we have*

$$a_n^{ord} \to \mathbb{1}(n \le \lambda^{-1} + 1), \tag{4.14}$$

where $\mathbb{1}(E)$ refers to the indicator function of the event E. □

Note that for channels supporting single packet reception, $\lambda > 1$ and hence only one packet is successfully received. However, in channels supporting multipacket reception, we can have $\lambda < 1$ [2, 11, 30]. If a_n^{ord} denotes the reception probability, then na_n^{ord} denotes the average number of successfully received packets. Thus, as long as the number of users contending for a particular carrier is less than a certain fixed upper limit, all the packets transmitted are guaranteed to be received successfully.

Let $T_{N_u}^{ord}$ denote the average number of packets received successfully per carrier. From (4.7), we can easily obtain that

$$T_{N_u}^{ord} = \sum_{k=1}^{N_u} a_k^{ord} \Pr\{n_j = k\}, \tag{4.15}$$

where $\Pr\{n_j = k\}$ is given by (4.6). We now study the behaviour of $T_{N_u}^{ord}$ in optimally, densely, and sparsely populated networks.

4.4.7 Densely Populated Networks

When the network is densely populated, following the lines of proof of Proposition 3, it can be shown that $T_{N_u}^{ord} \to 0$ as $N_u, N_c \to \infty$. Thus, in densely populated networks, the CAMCRA scheme with ordered selection offers no more throughput gain that the CAMCRA scheme with random selection.

4.4.8 Optimally Populated Networks

In Proposition 2, we have obtained the average number of successful packets for an optimally populated network following the CAMCRA scheme with random selection. For any population density α, we have proved that the maximum achievable throughput in the CAMCRA scheme is $\zeta(\alpha)$ given by (4.9). A similar analysis holds for networks following the CAMCRA scheme with ordered selection. In fact, substituting (4.13) in (4.9) we get the following result.

Proposition 13 *Consider an optimally populated slotted random access network with capture consisting of N_u users and N_c carriers. Let T_{N_u} given by (4.15) denote the average number of successfully received packets per carrier. Then for any $\alpha > 0$, we have*

$$T_{N_u}^{ord} \to \zeta^{ord}(\alpha) \tag{4.16}$$

where

$$\zeta^{ord}(\alpha) = \alpha e^{-\alpha} \sum_{k=0}^{K_0} \frac{\alpha^k}{k!} \tag{4.17}$$

and K_0 is the integer part of λ^{-1}. $\square$

4.4.9 Sparsely Populated Networks

When the network is sparsely populated, each user selects $c > 1$ *best* carriers to transmit packets. Then the probability that a particular carrier j is assigned to user 1 is $p = \frac{c}{N_c}$ and the average packet reception probability is given by (4.10). As discussed before, if for some $\alpha > 0$, each user chooses $c = \frac{N_c}{N_u}\alpha$ carriers, we find that $N_u \frac{c}{N_c} = \alpha$ and the network behaves like an optimally populated network with density α and attains a maximum throughput of $\zeta^{ord}(\alpha)$ given by (4.17). If $\zeta^{ord}(\alpha^*) = \sup_{\alpha>0} \zeta^{ord}(\alpha)$, then by an analysis similar to Proposition 5, we can conclude that $c = \frac{N_c}{N_u}\alpha^*$ maximizes the network throughput to $\zeta^{ord}(\alpha^*)$.

In Figure 12, assuming a value of $K_0 = 5$, we have plotted the asymptotic throughput curves of the CAMCRA scheme with random and ordered selection. As we can see from the figure, for low and moderate values of population density α, ordered selection yields a very high throughput compared to random selection. However, for large values of α,

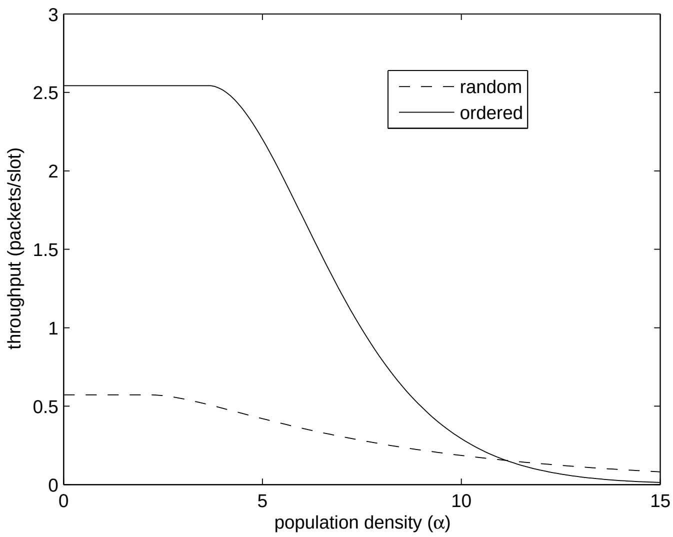

Figure 12: Throughput comparison for random access networks with capture following the CAMCRA scheme (a) Solid: Ordered Selection (b) Dotted: Random Selection.

random selection is seen to perform better than ordered selection. The phenomenon can be intuitively explained as follows: For a network with N_u users and N_c carriers following the CAMCRA scheme with ordered selection, it can be seen from (4.6) that the average number of users per carrier is $\alpha = \frac{N_u}{N_c}$. It is also important to note from Proposition 6 that as long as the number of users contending for a particular carrier is below a certain limit, the packets of all the users is successfully received. Otherwise, no packet is successfully received. When the population density α is low or moderate, since the average number of users per carrier is α, with very high probability all the users contending for a particular carrier successfully transmit their packets. When α is very high, then on an average the number of users per carrier is very high, and hence with very low probability, the users get to successfully transmit their packets.

4.5 Conclusion

In this chapter, we have studied the effect of CAMCRA scheme on the stability of random access networks with capture. In particular, we have shown that the proposed CAMCRA scheme has population independent transmission control and yet stabilizes capture networks. This is in contrast to the existing schemes which exhibit instability if the transmission control is unaware of the network population.

APPENDIX A

A.1 Proof of Proposition 1

If $\omega < 0$, we can write $a_n(\omega) = n^{\omega}\left(1 - \frac{1}{n^{1-\omega}}\right)^{n-1} < \frac{1}{n^{|\omega|}} \to 0$ as $n \to \infty$. Suppose $\omega > 0$. From (2.7), we note that $a_n(\omega)$ can be written as $a_n(\omega) = \frac{n^{\omega}{\alpha_n}^{n^{\omega}}}{\beta_n}$, where $\beta_n = 1 - \frac{1}{n^{1-\omega}}$ and $\alpha_n = \left(1 - \frac{1}{n^{1-\omega}}\right)^{n^{1-\omega}}$. We can see that $\beta_n \to 1$ as $n \to \infty$, and that α_n is a decreasing sequence with $\alpha_n \to e^{-1}$ as $n \to \infty$. In particular for sufficiently large n, $\alpha_n \leq \frac{2}{3} < 1$. Thus, for sufficiently large n, $n^{\omega}\alpha_n^{n^{\omega}} \leq n^{\omega}\left(\frac{2}{3}\right)^{n^{\omega}} \to 0$ as $n \to \infty$. It follows that $a_n(\omega) \to 0$ as $n \to \infty$. $\square$

A.2 Proof of Proposition 2

Consider a network with N_u users and N_c carriers following the CAMCRA scheme with channel uncertainty ω given by (2.2). In the CAMCRA scheme, we see from (4.6) that in any time slot the number of users in a particular carrier j is a random variable. Given that n users contend for carrier j, let $a'_n(\omega)$ denote the success probability. The following lemma gives the exact expression for $a'_n(\omega)$ and its asymptotic behaviour.

Lemma II.1: We have

$$a'_n(\omega) = n\hat{p}_n(1 - \hat{p}_n)^{n-1}, \tag{A.1}$$

where $\hat{p}_n = 1 - (1 - \delta_n^{1-\omega})^{N_c}$ and $\delta_n = 1 - \left(1 - \frac{1}{n}\right)^{\frac{1}{N_c}}$. Moreover, $a'_n(\omega) \to 0$ as $n \to \infty$.

Proof: For the i^{th} user, let $\hat{h}_{i,k} = h_{i,k} + e_{i,k}$ denote the instantaneous estimated channel gain on any carrier k. Since for any k, $\hat{h}_{i,k}$ is complex Gaussian, i.i.d., with zero mean and $E\{|\hat{h}_{i,k}|^2\} = \sigma^2_{act}$, we have that $|\hat{h}_{i,k}|^2$ is exponentially distributed with $\Pr\{|\hat{h}_{i,k}|^2 > h\} = e^{-\frac{h}{\sigma^2_{act}}}$. Suppose that n users contend for carrier j_0 and let $\hat{p}_n$ denote the probability that a particular user transmits in j_0. Without loss of generality, let users $1 \leq i \leq n$ contend for j_0. Since we consider the collision model, we can see that the probability of success is $a'_n(\omega) = n\hat{p}_n(1 - \hat{p}_n)^{n-1}$. From the description of the CAMCRA scheme, we note that for any $1 \leq i \leq n$, $|\hat{h}_{i,j_0}|^2 = \max_{1 \leq k \leq N_c} |\hat{h}_{i,k}|^2$ and hence $\Pr\{|\hat{h}_{i,j_0}|^2 > h\} = 1 - (1 - e^{-\frac{h}{\sigma^2_{act}}})^{N_c}$. If σ^2_{act} were known exactly, then every user i contending for j_0 chooses a threshold $\hat{H}_0(n)$, so that $\Pr\{|\hat{h}_{i,j_0}|^2 > \hat{H}_0(n)\} = \frac{1}{n}$ which gives us $\hat{H}_0(n) = -\sigma^2_{act} \log \left\{1 - \left(1 - \frac{1}{n}\right)^{\frac{1}{N_c}}\right\}$. However, since the estimated variance of the channel coefficient $\hat{h}_{i,k}$ is σ^2_{est}, each user chooses a threshold $\hat{H}'_0(n) = -\sigma^2_{est} \log \left\{1 - \left(1 - \frac{1}{n}\right)^{\frac{1}{N_c}}\right\} \triangleq -\sigma^2_{est} \log \delta_n$. Hence we have $\hat{p}_n = \Pr\{|\hat{h}_{i,j_0}|^2 > \hat{H}'_0(n)\} = 1 - (1 - e^{-\frac{\hat{H}'_0(n)}{\sigma^2_{act}}})^{N_c} = 1 - (1 - \delta_n^{1-\omega})^{N_c}$, from which (A.1) follows.

To prove that $a'_n(\omega) \to 0$, we proceed as follows. We have that $\delta_n = 1 - \left(1 - \frac{1}{n}\right)^{\frac{1}{N_c}} = \frac{1}{nN_c} + O\left(\frac{1}{n^2 N_c^2}\right)$ for large n and N_c. Therefore,

$$\hat{p}_n = 1-(1-\delta_n^{1-\omega})^{N_c} = 1-\left(1 - \frac{1}{(nN_c)^{1-\omega}}\left\{1 + O\left(\frac{1}{nN_c}\right)\right\}\right)^{N_c} + o(1) = 1-e^{-\frac{N_c}{(nN_c)^{1-\omega}}} + o(1).$$

Therefore we can obtain $a'_n(\omega) = n(1 - e^{-\frac{N_c}{(nN_c)^{1-\omega}}})e^{-(nN_c)^\omega} + o(1)$ which can be rewritten separately for $\omega > 0$ and $\omega < 0$ as

$$a'_n(\omega) = \begin{cases} ne^{-(nN_c)^\omega} + o(1) & \omega > 0 \\ \frac{1}{(nN_c)^{-\omega}} + o(1) & \omega < 0. \end{cases}$$

From the above equation, it is immediately obvious that $a'_n(\omega) \to 0$ as $n \to \infty$ for any $\omega \neq 0$. $\square$

Proof of Proposition 2: For any particular carrier j, let n_j denote the number of users contending in any particular time slot in that carrier. From (4.6) we find that

$$\Pr\{n_j = k\} = \begin{pmatrix} N_u \\ k \end{pmatrix} \left(\frac{1}{N_c}\right)^k \left(1 - \frac{1}{N_c}\right)^{N_u - k}. \tag{A.2}$$

From (A.2) and Lemma II.1, it follows that the average probability of success per slot in the CAMCRA scheme operating with uncertainty ω is

$$\zeta_{mc}(\alpha, \omega) = \sum_{k=1}^{N_u} a'_k(\omega) \Pr\{n_j = k\}. \tag{A.3}$$

To prove the statement in Proposition 2, we need to use the following lemma which can be found in pp. 37 – 39 of [45].

LEMMA II.2: Consider the binomial coefficient given by (A.2). Then as $N_u, N_c \to \infty$, with $\alpha = \frac{N_u}{N_c}$ fixed, we have

$$\Pr\{n_j = k\} = e^{-\alpha}\frac{\alpha^k}{k!}\left(1 + \frac{d_1 + d_2 k + d_3 k^2}{N_u}\right) + \frac{O(1)}{N_u^2}, \quad \text{(A.4)}$$

for some constants d_1, d_2, and d_3.$\square$

Substituting (C.6) in (A.3), and allowing $a'_k = a'_k(\omega)$, we get $\zeta_{mc}(\alpha,\omega) = \sum_{k=1}^{N_u} a'_k e^{-\alpha}\frac{\alpha^k}{k!} + \frac{B(N_u)}{N_u} + \frac{O(1)}{N_u^2}$, where $B(N_u) = \sum_{k=1}^{N_u} e^{-\alpha}\frac{\alpha^k}{k!} a'_k(d_1 + d_2 k + d_3 k^2)$. Since $a'_k < 1$, we get $B(N_u) \leq \sum_{k=1}^{N_u} e^{-\alpha}\frac{\alpha^k}{k!}(d_1 + d_2 k + d_3 k^2)$, for sufficiently large N_u. Since $e^{-\alpha}\sum_{k=1}^{N_u}\frac{\alpha^k}{k!}$, $e^{-\alpha}\sum_{k=1}^{N_u}\frac{k\alpha^k}{k!}$, and $e^{-\alpha}\sum_{k=1}^{N_u}\frac{k^2\alpha^k}{k!}$ exist and are of $O(1)$, we find that $\frac{B(N_u)}{N_u} + \frac{O(1)}{N_u^2} = \frac{O(1)}{N_u}$ and that

$$\zeta_{mc}(\alpha,\omega) = \sum_{k=1}^{N_u} a'_k e^{-\alpha}\frac{\alpha^k}{k!} + \frac{O(1)}{N_u}$$

from which (2.8) follows. $\square$

A.3 Proof of Proposition 3

Consider a network with N_u users and N_c carriers. To prove (2.15) and (2.16), we first note from (2.14)

$$\eta_{sc}(\alpha,\omega) = N_u(1-\hat{p}_{N_u})^{N_u-1}E\{\hat{R}\mathbb{1}\} = \underbrace{N_u\hat{p}_{N_u}(1-\hat{p}_{N_u})^{N_u-1}\log_2(1+B^2\ln N_u)}_{\mathcal{E}_1}$$
$$+\underbrace{N_u(1-\hat{p}_{N_u})^{N_u-1}(E\{\hat{R}\mathbb{1}\} - \hat{p}_{N_u}\log_2(1+B^2\ln N_u))}_{\mathcal{E}_2} \quad \text{(A.5)}$$

where $B^2 = 1$ if $\omega = 0$ and $B^2 = \sigma_{est}^2$ if $\omega \neq 0$.

If $\omega = 0$, then we have from (2.6) that $\hat{p}_{N_u} = \frac{1}{N_u}$ and hence $\mathcal{E}_1 = \left(1 - \frac{1}{N_u}\right)^{N_u-1}\log_2(1 + \sigma_{est}^2 \ln N_u)$.

If $\omega \neq 0$, then $\mathcal{E}_1 = N_u^{\omega}\left(1 - \frac{1}{N_u^{1-\omega}}\right)^{N_u-1}\log_2(1+\sigma_{est}^2\ln N_u)$. If $\omega < 0$, we immediately have $\mathcal{E}_1 < \frac{\log_2(1+\sigma_{est}^2\ln N_u)}{N_u^{-\omega}} = o(1)$. If $\omega > 0$, then using $1 - x < e^{-x}$, we have $\mathcal{E}_1 < N_u^{\omega}e^{-N_u^{\omega}}\log_2(1+\sigma_{est}^2\ln N_u) = o(1)$.

It remains to show that $\mathcal{E}_2 = o(1)$ for all ω. To that end we have,

$$E\{\hat{R}\mathbb{1}\} = \underbrace{\int_{\hat{H}'_0}^{\hat{H}'_0+H_1}\log_2(1+h)f(h)dh}_{\mathcal{D}_1} + \underbrace{\int_{\hat{H}'_0+H_1}^{\infty}\log_2(1+h)f(h)dh}_{\mathcal{D}_2}$$

where $f(h) = \frac{1}{\sigma_{act}^2} e^{-\frac{h}{\sigma_{act}^2}}$, $\hat{H}_0' = \sigma_{est}^2 \ln N_u$ and $H_1 = \sigma_{est}^2 \ln \ln N_u$. We have that

$$\begin{aligned} \mathcal{D}_2 &< \log_2(1 + \hat{H}_0' + H_1) \int_{\hat{H}_0' + H_1}^{\infty} f(h) dh \\ &= \frac{\log_2(1 + \sigma_{est}^2 \ln N_u + \sigma_{est}^2 \ln \ln N_u)}{N_u^{1-\omega} (\ln N_u)^{1-\omega}} < K_1 \frac{\log_2(1 + \ln N_u)}{N_u^{1-\omega} (\ln N_u)^{1-\omega}} \end{aligned} \tag{A.6}$$

for some sufficiently large constant K_1.

From the mean value theorem for integrals we have

$$\begin{aligned} \mathcal{D}_1 &= \int_{\hat{H}_0'}^{\hat{H}_0' + H_1} \log_2(1 + h) f(h) dh = \log_2(1 + H_0'') \int_{\hat{H}_0'}^{\hat{H}_0' + H_1} f(h) dh \\ &= \log_2(1 + H_0'') \left(\frac{1}{N_u^{1-\omega}} - \frac{1}{N_u^{1-\omega} (\ln N_u)^{1-\omega}} \right) \end{aligned} \tag{A.7}$$

for some $H_0'' \in (\hat{H}_0', \hat{H}_0' + H_1)$.

From (A.5) we find that

$$\mathcal{E}_2 = N_u \left(1 - \frac{1}{N_u^{1-\omega}} \right)^{N_u - 1} \left(\mathcal{D}_1 + \mathcal{D}_2 - \frac{1}{N_u^{1-\omega}} \log_2(1 + B^2 \ln N_u) \right).$$

Substituting (A.7) and (A.6), we find that

$$\mathcal{E}_2 < N_u^{\omega} \left(1 - \frac{1}{N_u^{1-\omega}} \right)^{N_u - 1} \left\{ \log_2 \left(\frac{1 + H_0''}{1 + B^2 \ln N_u} \right) + \frac{\log_2(1 + B^2 \ln N_u)}{(\ln N_u)^{1-\omega}} + \frac{\log_2(1 + H_0'')}{(\ln N_u)^{1-\omega}} \right\}.$$

If $\omega = 0$, since $H_0'' = \ln N_u + o(\ln N_u)$, each of the term within the brackets is $o(1)$. If $\omega \neq 0$, by an analysis similar to that of $\mathcal{E}_1$ and Proposition 1 we can show that each term is $o(1)$. □

A.4 Proof of Multiuser Diversity Theorem

Let $\{h_i\}_{i=1}^n$ and $\{e_i\}_{i=1}^n$ denote the instantaneous channel fading coefficients and the errors in the channel estimate of the n users in a particular carrier. From (2.17), we find that $E\{\hat{R}\} = E\{\log_2(1 + |h_M|^2)\}$, where $|h_M + e_M|^2 = \max_{1 \leq k \leq n} |h_k + e_k|^2$. Also we note that $|h_M + e_M| \leq |h_M| + |e_M| \leq |h_M| + E_{max}$, where $E_{max} = \max_{1 \leq k \leq n} |e_k|$ and $E\{|e_k|^2\} = \sigma_e^2$. Thus, we can write $\frac{|h_M|}{\sqrt{\ln n}} \geq f_n \stackrel{\Delta}{=} \frac{|h_M + e_M|}{\sqrt{\ln n}} - \frac{E_{max}}{\sqrt{\ln n}}$ from which we get

$$E\{\hat{R}\} \geq E\{\log_2(1 + f_n^2 \ln n)\}. \tag{A.8}$$

From [21], we can show that $\frac{|h_M + e_M|^2}{\ln n} \to \sigma_{act}^2$ and $\frac{E_{max}^2}{\ln n} \to \sigma_e^2$ in probability. Thus, $f_n \to A = \sigma_{act} - \sigma_e$ in probability. Hence, it can be easily shown that

$$S(n) = \log_2 \left(\frac{1 + f_n^2 \ln n}{1 + A^2 \ln n} \right)$$

converges to zero in probability.

Also from [1] pp. 73, we know that $E\{E_{max}^2\} = \sigma_e^2 \sum_{k=1}^{n} \frac{1}{k} = \sigma_e^2 \ln n + \sigma_e^2 \gamma + o(1)$, where $\gamma = 0.577...$ is the Euler's constant. Thus $E\left\{\frac{E_{max}^2}{\ln n}\right\} = \sigma_e^2 + o(1)$. Similarly, we have $E\left\{\frac{|h_M+e_M|^2}{\ln n}\right\} = \sigma_{act}^2 + o(1)$. It follows that $E\{|f_n|^4\} = E\left\{\frac{|h_M+e_M|^2}{\ln n}\right\} + E\left\{\frac{E_{max}^2}{\ln n}\right\} - 2E\left\{\frac{|h_M+e_M|E_{max}}{\ln n}\right\} \leq E\left\{\frac{|h_M+e_M|^2}{\ln n}\right\} + E\left\{\frac{E_{max}^2}{\ln n}\right\} + 2\sqrt{E\left\{\frac{|h_M+e_M|^2}{\ln n}\right\} E\left\{\frac{E_{max}^2}{\ln n}\right\}} = (\sigma_{act} + \sigma_e)^2 + o(1)$. Since $\sup_n E\{|f_n^4|\} < \infty$, it follows that $|f_n|^2$ is uniformly integrable (u.i.). Hence given $\epsilon > 0$, there exists M sufficiently large s.t. for all n

$$E\{|f_n|^2 \mathbb{1}(|f_n| > M)\} < \epsilon \quad and \quad \Pr\{|f_n| > M\} < \epsilon,$$

where $\mathbb{1}(B)$ refers to the indicator function of set B [34]. Now,

$$E\{S(n)\} = \underbrace{E\{S(n)\mathbb{1}(|f_n| > M)\}}_{\mathcal{E}_1} + \underbrace{E\{S(n)\mathbb{1}(|f_n| \leq M)\}}_{\mathcal{E}_2}.$$

Now if $|f_n| \leq M$, it can be easily shown that there exists $B_1 > 0$ s.t. $S(n) \leq B_1$ for all n. And since $S(n) \to 0$ in probability, it follows from Bounded Convergence Theorem [34] that $\mathcal{E}_2 = E\{S(n)\mathbb{1}(|f_n| \leq M)\} \to 0$ as $n \to \infty$. Now, using the fact that $\ln(1+x) \leq x$ for all $x > 0$, we get

$$\begin{aligned}
\mathcal{E}_1 &= E\{S(n)\mathbb{1}(|f_n| > M)\} = E\{\log_2\left(1 + \frac{(f_n^2 - A^2)\ln n}{1 + A^2 \ln n}\right)\mathbb{1}(|f_n| > M)\} \\
&\leq \log_e 2 E\left\{\frac{f_n^2 - A^2}{\frac{1}{\ln n} + A^2}\mathbb{1}(|f_n| > M)\right\} \leq B_1 E\{f_n^2 \mathbb{1}(|f_n| > M)\} + B_2 \Pr\{|f_n| > M\} \leq B_3 \epsilon
\end{aligned}$$

for some sufficiently large constants $B_1, B_2, B_3 > 0$. This implies that $\mathcal{E}_1 \to 0$ as $n \to 0$ implying that $E\{S(n)\} = \mathcal{E}_1 + \mathcal{E}_2 = o(1)$. Therefore from (A.8) we find that $E\{\hat{R}\} \geq E\{\log_2(1 + f_n^2 \ln n)\} = E\{S(n)\} + \log_2(1 + A^2 \ln n) = \log_2(1 + A^2 \ln n) + o(1)$. $\square$

A.5 Proof of Proposition 4

Let $\hat{h}_{i,j}$ denote the channel estimate of the i^{th} user on the j^{th} carrier. In the CAMCRA scheme we note from the description that the number of users contending per carrier varies from slot to slot. If n_j denotes the number of carriers in a particular slot in carrier j, then $\Pr\{n_j = k\}$ is given by (4.6). By an arguement similar to the derivation of (3.1) and (2.13) we can see that

$$\eta_{mc}(\alpha,\omega) = \sum_{n=1}^{N_u} \Pr\{n_j = n\} n(1-\hat{p}_n)^{n-1} E\left\{R(|\hat{h}_{m(1),1}|^2)\mathbb{1}(|\hat{h}_{m(1),1}|^2 > \hat{H}_0')\right\} \tag{A.9}$$

where $m(1) = \arg\max_{1\le i\le N_u}\{|h_{i,1}+e_{i,1}|^2\}$ and $\hat{H}_0'$ is the threshold chosen by each user. Using Lemma II.2 and shortened notations, we have

$$\eta_{mc}(\alpha,\omega) = e^{-\alpha}\sum_{n=1}^{N_u} \frac{\alpha^n}{n!} n(1-\hat{p}_n)^{n-1} E\{\hat{R}\mathbb{1}\} = e^{-\alpha}\sum_{n=1}^{N_u} \frac{\alpha^n}{n!} n\hat{p}_n(1-\hat{p}_n)^{n-1} E\{\hat{R}\} + \mathcal{I}_1 + \mathcal{I}_2$$

where $\mathcal{I}_1 = e^{-\alpha}\sum_{n=1}^{N_u} \frac{\alpha^n}{n!} n\hat{p}_n(1-\hat{p}_n)^{n-1}(E\{\mathbb{1}\}E\{\hat{R}\} - E\{\hat{R}\mathbb{1}\})$ and
$\mathcal{I}_2 = e^{-\alpha}\sum_{n=1}^{N_u}\left(\frac{\alpha^n}{n!}\frac{(d_1+d_2n+d_3n^2)}{N_u} + \frac{O(1)}{N_u^2}\right)E\{\hat{R}\mathbb{1}\}$. It remains to prove that $\mathcal{I}_1 = o(1) = \mathcal{I}_2$. First, similar to proof of Proposition 3, it can be shown that $|E\{\mathbb{1}\}E\{\hat{R}\} - E\{\hat{R}\mathbb{1}\}| = o(1)$ independent of n and hence immediately $\mathcal{I}_1 = o(1)$.
Secondly, we note that $E\{\hat{R}\mathbb{1}\} = E\left\{R(|\hat{h}_{m(1),1}|^2)\mathbb{1}(|\hat{h}_{m(1),1}|^2 > \hat{H}_0')\right\} < E\{R(|\hat{h}_{m(1),1}|^2)\}$ $< E\{\max_{1\le i\le N_u, 1\le j\le N_c} R(|\hat{h}_{i,j}|^2)\} = \log_2(1+\sigma_{act}^2 \ln N_u N_c) + o(1) < B\log_2(1+\ln N_u) + o(1)$ for some sufficiently large constant $B > 0$. Therefore $\mathcal{I}_2 < B_1 \frac{\ln\ln N_u}{N_u} = o(1)$ for some sufficiently large constant $B_1 > 0$.
To prove (2.19), we use the lower bound of MDT to obtain

$$\begin{aligned}
\eta_{mc}(\alpha,\omega) &= e^{-\alpha}\sum_{n=1}^{N_u}\frac{\alpha^n}{n!}a_n' E\{\hat{R}\} + o(1) \\
&\ge e^{-\alpha}\sum_{n=1}^{N_u}\frac{\alpha^n}{n!}a_n' \log_2(1+A^2\ln nN_c) + o(1) \\
&\ge \log_2\left(1 + A^2 \ln\frac{N_u}{\alpha}\right) e^{-\alpha}\sum_{n=1}^{\infty}\frac{\alpha^n}{n!}a_n' + o(1). \ \square
\end{aligned}$$

A.6 Proof of Proposition 5

Consider a network with N_u users and N_c carriers with the opportunistic splitting algorithm being applied on each carrier. First, we let the uncertainty $\omega = 0$. In the NS-RA scheme we notice that all the N_u users contend in each carrier. If the random variable X_{N_u} denotes the number of slots required to resolve the collision in a particular carrier j, then from Lemma 1 of [53] we know that $E\{X_{N_u}\} \le \log_2 N_u + 1$ which implies that $X_{N_u} < \infty \quad a.s.$ Moreover, if N_t minislots are totally allocated for collision resolution, then we see that the average success probability is $\zeta'_{sc}(\alpha, 0) = \Pr\{X_{N_u} \leq N_t\} > 1 - \epsilon$ for any $\epsilon \in (0, 1)$ and for sufficiently large N_t.

We now let the uncertainty $\omega \neq 0$. For carrier j, let $\{|\hat{h}_{i,j}|^2\}_{i=1}^{N_u}$ denote the estimated channel gains of the users. From the description of the opportunistic algorithm we find that at level k, the real line is split into intervals of the form $[\sigma_{est}^2 \ln(a_{k,l} N_u), \sigma_{est}^2 \ln(b_{k,l} N_u)]$ where $a_{k,l}$ and $b_{k,l}$ are the threshold values for particular paths in the splitting tree. Thus for example if p_k denotes the probability that the splitting algorithm is able to resolve the collisions only at minislot k, then for $k = 1, 2$ we can write

$$p_1 = n \frac{1}{n^{1-\omega}} \left(1 - \frac{1}{n^{1-\omega}}\right)^{n-1}$$

and

$$\begin{aligned} p_2 &= n \frac{2^{1-\omega} - 1}{n^{1-\omega}} \left(1 - \frac{2^{1-\omega}}{n^{1-\omega}}\right)^{n-1} + n \frac{1}{(2n)^{1-\omega}} \left(\left(1 - \frac{1}{(2n)^{1-\omega}}\right)^{n-1} - \left(1 - \frac{1}{n^{1-\omega}}\right)^{n-1} \right) \\ &= A_{2,1} n^{\omega} \left(1 - \frac{B_{2,1}}{n^{1-\omega}}\right)^{n-1} + A_{2,2} n^{\omega} \left(1 - \frac{B_{2,2}}{n^{1-\omega}}\right)^{n-1} + A_{2,3} n^{\omega} \left(1 - \frac{B_{2,3}}{n^{1-\omega}}\right)^{n-1} \end{aligned}$$

where the quantities $A_{2,1}, B_{2,1}, A_{2,2}, B_{2,2}, A_{2,3}$, and $B_{2,3}$ are all $O(1)$. Continuing this way, we can see that

$$\zeta'_{sc}(\alpha, \omega) = \Pr\{X_{N_u} \leq N_t\} = \sum_{k=1}^{N_t} p_k = \sum_{k=1}^{N_t} \sum_{j=1}^{2^k - 1} A_{k,j} n^{\omega} \left(1 - \frac{B_{k,j}}{n^{1-\omega}}\right)^{n-1}.$$

For any finite N_t, from Proposition 1, we note that each term in the summation goes to zero asymptotically and hence (2.20) follows. □

A.7 Proof of Proposition 6

Let p_n^{rs} denote the probability of success of the reservation scheme proposed in Section 2.7 when n users contend in a particular carrier. For $i = 1, 2, ...n$, let $X_i \in \{0, 1, ...N_t - 1\}$ be the random variable denoting the number chosen by user i. Successful reservation occurs if $\min\{X_i\}$ is unique. Note that it is only a sufficient and not a necessary condition. Hence $p_n^{rs} \geq n \Pr\{X_2 > X_1, ..., X_n > X_1\}$ which can be evaluated as

$$p_n^{rs} \geq \frac{n}{N_t} \sum_{k=1}^{N_t - 1} \left(\frac{k}{N_t}\right)^{n-1}. \tag{A.10}$$

In the CAMCRA scheme with reservation, each user contending in a particular carrier chooses to ask for reservation if and only if its threshold is greater than a predetermined threshold H_0. Clearly the success probability would be reduced if all the users contending ask for reservation. If n_j denotes the number of carriers contending for carrier j, then the probability of success can then be lower bounded as

$$\zeta'_{mc}(\alpha, \omega) \geq \sum_{n=1}^{N_u} \Pr\{n_j = n\} p_n^{rs}.$$

For asymptotics, we use Lemma II.1 and (A.10) and rewrite as

$$\begin{aligned}
\zeta'_{mc}(\alpha, \omega) &\geq e^{-\alpha} \sum_{n=1}^{\infty} \frac{\alpha^n}{n!} p_n^{rs} + o(1) \\
&\geq e^{-\alpha} \sum_{n=1}^{\infty} \frac{n}{N_t} \frac{\alpha^n}{n!} \sum_{k=1}^{N_t - 1} \left(\frac{k}{N_t}\right)^{n-1} + o(1) = \frac{\alpha}{N_t} \frac{(1 - e^{\frac{\alpha}{N_t} - \alpha})}{(e^{\frac{\alpha}{N_t}} - 1)} - o(1). \quad \square
\end{aligned}$$

APPENDIX B

B.1 Proof of Theorem 2

For simplicity, let n denote the total number of carriers and $\{H_k\}_{k=1}^n$ the channel gains. Each of the random variable H_k is exponentially distributed with unit mean unit variance. We arrange H_k in decreasing order and label them as G_k. We now prove Theorem 2 through a series of Propositions.

Proposition A.1: For any $0 < \alpha < 1$ and for any $1 \leq k \leq n\alpha$, let $\mu_k = E\{G_k\}$ denote the mean of the random variable G_k. Then for sufficiently large n,

$$E\{\log_2(1 + G_k)\} = \log_2(1 + \mu_k) + o(1),$$

where the $o(1)$ term is independent of k.

Proof: It is well known (see [1], pp.73) that $\mu_k = E\{G_k\} = \sum_{l=k}^n \frac{1}{l}$ and $\sigma_k^2 = E\{(G_k - \mu_k)^2\} = \sum_{l=k}^n \frac{1}{l^2}$. It is also known that $H_n = \sum_{l=1}^n \frac{1}{l} = \ln n + \gamma + o(1)$, where $\gamma = 0.577...$ is the Euler's constant and $\sum_{l=1}^n \frac{1}{l^2} \to \frac{\pi^2}{6}$ as $n \to \infty$. Now if $k > k_0 = \sqrt{n}$, it is easy to see that $\mu_k > \mu_{n\alpha} \sim \ln\left(\frac{n}{n\alpha}\right) = -\ln\alpha$ and $\sigma_k^2 < \sigma_{k_0}^2 \sim \int_{\sqrt{n}}^n \frac{1}{x^2}dx = o(1)$. Thus if $\sqrt{n} < k < n\alpha$, $\frac{\sigma_k^2}{\mu_k^2}$ can be made *uniformly* arbitrarily small. If $k < k_0 = \sqrt{n}$, $\sigma_k^2 < \sigma_1^2 \sim \frac{\pi^2}{6}$ and $\mu_k > \mu_{k_0} \sim \ln\left(\frac{n}{\sqrt{n}}\right)$. Thus, if $k < \sqrt{n}$, $\frac{\sigma_k^2}{\mu_k^2}$ can be made *uniformly* arbitrarily small. Thus, given any $\epsilon > 0$, for all sufficiently large n, $\sup_{k=1}^{n\alpha} \frac{\sigma_k^2}{\mu_k^2} < \epsilon$. If we denote $f_k = \frac{G_k}{\mu_k}$, it then follows that f_k are uniformly integrable (u.i.) and hence for any $\epsilon > 0$, there exists $M > 1$ s.t. for any n and for all $1 \leq k \leq n\alpha$,

$$E\{|f_k|\mathbb{1}(|f_k| > M)\} < \epsilon \quad and \quad \Pr\{|f_k| > M\} < \epsilon.$$

Now, for any $\epsilon > 0$,

$$\Pr\left\{\left|\frac{G_k}{\mu_k} - 1\right| \geq \epsilon\right\} \leq \frac{\sigma_k^2}{\mu_k^2\epsilon^2} \to 0$$

as $n \to \infty$. Thus, for any $k = 1, 2, ...n\alpha$, $\frac{G_k}{\mu_k} \to 1$ and hence $S(k) = \log_2\left(\frac{1+G_k}{1+\mu_k}\right) \to 0$, in probability, uniformly. Now,

$$E\{S(k)\} = \underbrace{E\{S(k)\mathbb{1}(|f_k| > M)\}}_{\mathcal{E}_1} + \underbrace{E\{S(k)\mathbb{1}(|f_k| \leq M)\}}_{\mathcal{E}_2}.$$

If $|f_k| \leq M$, then it can be shown that $S(k)$ is uniformly bounded for all $1 \leq k \leq n\alpha$. And since $S(k) \to 0$ in probability, it follows from Bounded Convergence Theorem that $\mathcal{E}_2 = o(1)$.

Now, using the fact that $\ln(1+x) \leq x$ for all $x > 0$, we get

$$\mathcal{E}_1 \quad \leq \quad (\log_e 2)E\left\{\frac{|f_k - 1|}{\frac{1}{\mu_k}+1}\mathbb{1}(|f_k| > M)\right\} \leq B_1 E\{|f_k|\mathbb{1}(|f_k| > M)\} + B_2 \Pr\{|f_k| > M\} \leq B_3\epsilon$$

for some sufficiently large constants $B_1, B_2, B_3 > 0$. This implies that $\mathcal{E}_1 = o(1)$ and that $E\{S(k)\} = \mathcal{E}_1 + \mathcal{E}_2 = o(1)$. $\square$

From Proposition A.1 it follows that $C(\alpha)$ as defined in (3.10), can also be written as

$$C(\alpha) = \limsup_{n\to\infty} \frac{\sum_{k=1}^{n\alpha} \log_2(1+\mu_k)}{nC_{max}},$$

where $C_{max} = E\{\log_2(1+H_1)\} = \int_0^\infty \log_2(1+x)e^{-x}dx$. We have the following proposition.

Proposition A.2: For any $0 < \alpha < \alpha' < 1$,

$$\frac{\log_2(1-\ln\alpha)}{C_{max}} \geq \frac{C(\alpha') - C(\alpha)}{\alpha' - \alpha} \geq \frac{\log_2(1-\ln\alpha')}{C_{max}}. \tag{B.1}$$

Proof: From Proposition A.1, we have

$$E\{\log_2(1+G_k)\} = \log_2(1+\mu_k) + o(1) = \log_2(1+H_n - H_{k-1}) + o(1),$$

where $H_n = \sum_{k=1}^n \frac{1}{k} = \ln n + \gamma + o(1)$ where $\gamma = 0.577$ is the Euler's constant. Thus, given $0 < \alpha < \alpha' < 1$, we have that $\mu_{n\alpha} \sim -\ln\alpha > -\ln\alpha' \sim \mu_{n\alpha'}$. From Proposition A.2,

$$C(\alpha') - C(\alpha) = \limsup_{n\to\infty} \frac{\sum_{k=1}^{n\alpha'} \log_2(1+\mu_k)}{nC_{max}} - \limsup_{n\to\infty} \frac{\sum_{k=1}^{n\alpha} \log_2(1+\mu_k)}{nC_{max}}.$$

For any two sequences $\{a_n\}$ and $\{b_n\}$, it is a well known fact that if $\lim(a_n)$ exists, then $\limsup(a_n + b_n) = \lim(a_n) + \limsup(b_n)$. Thus,

$$\begin{aligned} C(\alpha') - C(\alpha) &= \lim_{n\to\infty} \frac{\sum_{k=n\alpha+1}^{n\alpha'} \log_2(1+\mu_k)}{nC_{max}} \\ &\geq \lim_{n\to\infty} \frac{n(\alpha'-\alpha)\log_2(1+\mu_{n\alpha'})}{nC_{max}} = \frac{(\alpha'-\alpha)}{C_{max}}\log_2(1-\ln\alpha'). \end{aligned}$$

The other part of the inequality can be shown similarly. $\square$

Allowing $\alpha' \to \alpha$ in (B.1) we obtain $C'(\alpha) = \frac{\log_2(1-\ln\alpha)}{C_{max}}$ and hence (3.11) follows.

Since the G_k's are ordered random variables arranged in decreasing order, it follows that $\log_2(1+G_k)$ are also arranged in decreasing order and $\sum_{k=1}^{n\alpha}\log_2(1+G_k) \geq \sum_{k=1}^{n\alpha}\log_2(1+H_k)$ or $C(\alpha) \geq \alpha$. Defining $g(\alpha) = C(\alpha) - \alpha$, it can be deduced that $g''(\alpha) < 0$ for all $0 < \alpha < 1$ implying that g is a non-zero concave function on $(0,1)$ and thus $C(\alpha) > \alpha$. $\square$

B.2 Appendix B.2: Proof of Claim in Theorem 3

Let $\alpha = \alpha_1\alpha_2$, $C(\alpha_1) = C_1, C(\alpha_2) = C_2$, and $C(\alpha_1\alpha_2) = C_{12}$. The three equations can be rewritten as

$$C_1 - C_{12} = \gamma^2, \tag{B.2}$$

$$C_2 - C_{12} = (1-\gamma)^2, \tag{B.3}$$

and

$$\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_{12}}. \tag{B.4}$$

From (B.4), we find that $C_1 - C_{12} = \frac{C_1 C_{12}}{C_2}$ and that $C_2 - C_{12} = \frac{C_2 C_{12}}{C_1}$ so that (B.2) and (B.3) can be rewritten as $\frac{C_1 C_{12}}{C_2} = \gamma^2$ and $\frac{C_2 C_{12}}{C_1} = (1-\gamma)^2$ which gives us $C_{12} = C(\alpha) = \gamma(1-\gamma)$. For a given γ, α can be determined uniquely as $\alpha = C^{-1}(\gamma(1-\gamma))$. Hence we need to solve for α_1 alone since $\alpha_2 = \frac{\alpha}{\alpha_1}$. Since $\frac{C_1}{C_2} = \frac{\gamma}{1-\gamma}$, we have

$$\frac{C(\alpha_1)}{C(\alpha/\alpha_1)} = \frac{\gamma}{1-\gamma}. \tag{B.5}$$

As function of α_1, with $\alpha \leq \alpha_1 \leq 1$, it can be seen that $\frac{C(\alpha_1)}{C(\alpha/\alpha_1)}$ is an invertible function with range $[C(\alpha), \frac{1}{C(\alpha)}]$. Since $C(\alpha) = \gamma(1-\gamma)$ and $\gamma < 1$, we find that $\frac{\gamma}{1-\gamma} \in [C(\alpha), \frac{1}{C(\alpha)}]$ and hence (B.5) has a unique solution. $\square$

APPENDIX C

C.1 Proof of Proposition 7

We need to show that $a_n \to 0$ as $n \to \infty$, where a_n is given by (4.5). Let $T_n = \frac{\sum_{j=1}^n h_j}{n}$ with $G_n(t) = \Pr\{T_n < t\}$. Then, for any $\epsilon > 0$, (4.5) can be rewritten as

$$\begin{aligned} a_n &= \Pr\left\{h_{\max} > \frac{nT_n + N_0}{1+\lambda^{-1}}\right\} \\ &= \int_0^\infty \left(1 - \left\{1 - F\left(\frac{nt + N_0}{1+\lambda^{-1}}\right)\right\}^n\right) dG_n(t), \\ &= \int_0^\epsilon \{1-(1-F)^n\} dG_n + \int_\epsilon^\infty \{1-(1-F)^n\} dG_n \\ &\triangleq \mathcal{I}_1 + \mathcal{I}_2, \end{aligned}$$

where $F(h) = \Pr\{h_1 > h\}$.

Since $E\{h_1\} < \infty$, by the Strong Law of Large Numbers (SLLN) (pp. 133 in [25]), we have $T_n \to \Delta \triangleq E\{h_1\}$ $a.s$ and hence $G_n(t) \to \mathbb{1}_{\{t>\Delta\}}$ where $\mathbb{1}_{\{.\}}$ refers to the indicator function. Therefore for any $0 < \epsilon < \frac{\Delta}{2}$, $\Pr\{T_n < \epsilon\} < \epsilon$ for n sufficiently large. For $\mathcal{I}_1$ we then have,

$$\begin{aligned} \mathcal{I}_1 &= \int_0^\epsilon \{1-(1-F)^n\} dG_n < 2\int_0^\epsilon dG_n \\ &= 2\Pr\{T_n < \epsilon\} < 2\epsilon. \end{aligned} \tag{C.1}$$

We now consider $\mathcal{I}_2$. First, we observe that for $t > \epsilon$,

$$F\left(\frac{nt + N_0}{1+\lambda^{-1}}\right) < F\left(\frac{n\epsilon + N_0}{1+\lambda^{-1}}\right) < F(nB\epsilon) \tag{C.2}$$

for some sufficiently large constant $B > 0$. It is well-known (pp. 46 in [25]) that if $E\{h_1\} < \infty$,

$$n\Pr\{h_1 > nB\epsilon\} \to 0$$

as $n \to \infty$ for any $\epsilon > 0$. Thus, $nF(nB\epsilon) < \epsilon$ for sufficiently large n and from (C.2) we have

$$\begin{aligned}
\mathcal{I}_2 &= \int_\epsilon^\infty \left(1 - \left\{1 - F\left(\frac{nt + N_0}{1 + \lambda^{-1}}\right)\right\}^n\right) dG_n(t) \\
&< \int_\epsilon^\infty \{1 - (1 - F(nB\epsilon))^n\} dG_n(t) < \int_\epsilon^\infty nF(nB\epsilon) dG_n(t) \\
&< nF(nB\epsilon) < \epsilon. \qquad \text{(C.3)}
\end{aligned}$$

From (C.1) and (C.3), we get that $a_n < 3\epsilon$ for sufficiently large n and the proof is complete. □

C.2 Proof of Proposition 8

For any particular carrier j let n_j denote the number of users contending in any particular time slot in that carrier. From (4.6) and (4.7), respectively, we find that

$$\Pr\{n_j = k\} = \begin{pmatrix} N_u \\ k \end{pmatrix} \left(\frac{1}{N_c}\right)^k \left(1 - \frac{1}{N_c}\right)^{N_u - k} \qquad \text{(C.4)}$$

and that the average number of packets successfully received per slot is

$$T_{N_u} = \sum_{k=1}^{N_u} a_k \Pr\{n_j = k\}. \qquad \text{(C.5)}$$

To prove that T_{N_u} converges to $\zeta(\alpha)$, i.e. to prove (4.8), we need to use the following lemma which can be found in pp. 37 – 39 of [45].

Lemma II.1: Consider the binomial coefficient given by (C.4). Then as $N_u, N_c \to \infty$, with $\alpha = \frac{N_u}{N_c}$ fixed, we have

$$\Pr\{n_j = k\} = e^{-\alpha}\frac{\alpha^k}{k!}\left(1 + \frac{d_1 + d_2 k + d_3 k^2}{N_u}\right) + \frac{O(1)}{N_u^2}, \qquad \text{(C.6)}$$

for some constants d_1, d_2 and d_3. □

Substituting (C.6) in (C.5), we get

$$\begin{aligned}
T_{N_u} &= \sum_{k=1}^{N_u} a_k \Pr\{n_j = k\} \\
&= \sum_{k=1}^{N_u} a_k e^{-\alpha}\frac{\alpha^k}{k!} + \frac{B(N_u)}{N_u} + \frac{O(1)}{N_u^2}.
\end{aligned}$$

where $B(N_u) = \sum_{k=1}^{N_u} e^{-\alpha}\frac{\alpha^k}{k!} a_k (d_1 + d_2 k + d_3 k^2)$. We know from Proposition 1 that $a_k \to 0$ as $k \to \infty$. Hence there exists a constant $M > 0$ such that $a_k \leq M$. Thus $B(N_u) \leq$

$M\sum_{k=1}^{N_u} e^{-\alpha}\frac{\alpha^k}{k!}(d_1+d_2k+d_3k^2)$, for sufficiently large N_u. Since $e^{-\alpha}\sum_{k=1}^{N_u}\frac{\alpha^k}{k!}$, $e^{-\alpha}\sum_{k=1}^{N_u}\frac{k\alpha^k}{k!}$, and $e^{-\alpha}\sum_{k=1}^{N_u}\frac{k^2\alpha^k}{k!}$ exist and are of $O(1)$, we find that $\frac{B(N_u)}{N_u}+\frac{O(1)}{N_u^2}=\frac{O(1)}{N_u}$ and that

$$T_{N_u}=\sum_{k=1}^{N_u} a_k e^{-\alpha}\frac{\alpha^k}{k!}+\frac{O(1)}{N_u}$$

from which (4.9) follows. □

C.3 Proof of Proposition 9

In densely populated networks, the number of users N_u far exceeds the number of carriers N_c, i.e., $N_u = o(N_c)$. Therefore as $N_u, N_c \to \infty$, $\alpha = \frac{N_u}{N_c} \to \infty$. We know from Proposition 2 that $T_{N_u} \to \zeta(\alpha)$ where $\zeta(\alpha)$ is given by (4.9). It is easy to see that $\lim_{\alpha\to\infty}\zeta(\alpha) = 0$. *However*, we cannot let $\alpha \to \infty$ in Proposition 2 to prove Proposition 3. We proceed as follows.

As before, let n_j denote the number of users contending for the j^{th} carrier in any particular time slot. Then we have from (4.6) that

$$\Pr\{n_j = k\} = \begin{pmatrix} N_u \\ k \end{pmatrix}\left(\frac{1}{N_c}\right)^k\left(1-\frac{1}{N_c}\right)^{N_u-k}. \tag{C.7}$$

When the network is densely populated, we have that $N_c = o(N_u)$ and hence

$$\frac{N_u}{N_c}\left(1-\frac{1}{N_c}\right) \to \infty \tag{C.8}$$

as $N_u, N_c \to \infty$. Under such a condition, it is well known [45] that the probability distribution given by (C.7) is "well-approximated" by a normal distribution with a mean

$$\mu = \frac{N_u}{N_c} \tag{C.9}$$

and a variance

$$\sigma^2 = \frac{N_u}{N_c}\left(1-\frac{1}{N_c}\right). \tag{C.10}$$

In fact, we have the following Lemma (pp. 82, Theorem 4 in [45]).

Lemma III.1: For the binomial probability distribution given by (C.7), let $N_c = o(N_u)$, i.e., let (C.8) be satisfied. Define

$$\mathcal{N}(k) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(k-\mu)^2}{2\sigma^2}} \tag{C.11}$$

where μ and σ^2 are given by (C.9) and (C.10), respectively. Then we have

$$\sum_{k=0}^{N_u} |\Pr\{n_j = k\} - \mathcal{N}(k)| = \frac{c_0}{\sigma}(1 + o(1)), \tag{C.12}$$

where $c_0 = \frac{1+4e^{-3/2}}{3\sqrt{(2\pi)}}$. $\square$

In other words, the "distance" between the normal distribution and the binomial distribution becomes arbitrarily close.

In a densely populated network, let the average number of packets received per slot be T_{N_u} given by (4.7). We know from Proposition 1 that the sequence a_k in (4.7) satisfies $a_k \to 0$ as $k \to \infty$. Hence there exists a constant $M > 0$ such that $a_k \leq M$. Then we have

$$\begin{aligned}
T_{N_u} &= \sum_{k=1}^{N_u} a_k \Pr\{n_j = k\} \\
&\leq \sum_{k=1}^{N_u} a_k |\Pr\{n_j = k\} - \mathcal{N}(k)| + \sum_{k=1}^{N_u} a_k \mathcal{N}(k) \\
&\leq M \sum_{k=0}^{N_u} |\Pr\{n_j = k\} - \mathcal{N}(k)| + \sum_{k=1}^{N_u} a_k \mathcal{N}(k) && \text{(C.13)} \\
&= M \frac{c_0}{\sigma}(1 + o(1)) + \delta_k && \text{(C.14)} \\
&= M o(1) + \delta_k && \text{(C.15)}
\end{aligned}$$

where (C.14) follows from (C.13) due to Lemma III.1 and (C.15) follows from (C.14) due to the fact that $\sigma^2 = \frac{N_u}{N_c}\left(1 - \frac{1}{N_c}\right) \to \infty$, as $N_u, N_c \to \infty$. If we can show that $\delta_k = o(1)$, then the proof is complete.

Since $a_k \to 0$, for any $\epsilon > 0$, there exists N_ϵ sufficiently large so that for all $k > N_\epsilon$, $a_k < \epsilon$. Therefore, we have

$$\begin{aligned}
\delta_k &= \sum_{k=1}^{N_u} a_k \mathcal{N}(k) \\
&= \sum_{k=1}^{N_\epsilon} a_k \mathcal{N}(k) + \sum_{k=N_\epsilon+1}^{N_u} a_k \mathcal{N}(k) \\
&\leq M \sum_{k=1}^{N_\epsilon} \mathcal{N}(k) + \epsilon \sum_{k=N_\epsilon+1}^{N_u} \mathcal{N}(k) \\
&\leq \frac{M N_\epsilon}{\sigma\sqrt{2\pi}} + \epsilon < 2\epsilon
\end{aligned}$$

for sufficiently large N_u. $\square$

C.4 Proof of Proposition 11

Let,

$$\zeta(\alpha) = e^{-\alpha} \sum_{k=1}^{\infty} \frac{\alpha^k a_k}{k!}. \tag{C.16}$$

and the sequence a_k have the following properties: a) $\lim_{k\to\infty} a_k = a \geq 0$, b) $a_k \geq a$, $\forall\, k$ and c) $\exists\, n_0$ s.t. $a_{n_0} > a$. We first prove the following lemma:

Lemma IV.1: $\exists\, \alpha_1 < \infty$ s.t. $\zeta(\alpha_1) > a$.

Proof: Suppose $\zeta(\alpha) \leq a$ for all α. Since $a_n < a$ for all n, we have that

$$\zeta(\alpha) - a = \sum_{n=1}^{\infty} \frac{\alpha^n}{n!} a_n - a = e^{-\alpha}\left(-a - \sum_{n=1}^{\infty} \frac{\alpha^n}{n!}(a - a_n)\right) < 0$$

which is a contradiction for $\alpha > \left(\frac{an_0!}{a_{n_0}-a}\right)^{\frac{1}{n_0}}$. □

Proof of Proposition 5(a1): Since from Lemma $IV.1$ we have that $\exists\, \alpha_1 < \infty$ s.t. $\zeta(\alpha_1) > a$,

$$\zeta^* = \sup_{\alpha>0} \zeta(\alpha) \geq \zeta(\alpha_1) > a.\ \square \tag{C.17}$$

Proof of Theorem 5(b1): Since $\lim_{\alpha\to\infty} \zeta(\alpha) = a$ it follows that that if (C.17) holds, $\exists\ \alpha^* < \infty$ s.t. $\zeta(\alpha^*) = \zeta^*$. □

Proof of Theorem 5(c1): We have,

$$\begin{aligned} \zeta(\alpha) - a &= e^{-\alpha} \sum_{n=1}^{\infty} \frac{\alpha^n}{n!} a_n - a \\ &= e^{-\alpha} \sum_{n=1}^{\infty} \frac{\alpha^n}{n!} (a_n - a) - a. \end{aligned}$$

Define $g(\alpha) = \sum_{n=1}^{\infty} \frac{\alpha^n}{n!} b_n - a$, where $b_n = a_n - a \geq 0\ \forall\, n$. Let $\beta_0 = \left(\frac{(a+1)n_0!}{a_{n_0}-a}\right)^{1/n_0}$. It is easy to show that the function g is continuous in $[0, \beta_0]$. Also, $g(0) = -a < 0$. It can be shown easily that $g(\beta_0) > 0$. Hence by Bolzano's Theorem (pp. 153 in [5]), $\exists\, \alpha_0 \in [0, \beta_0]$ s.t. $g(\alpha_0) = 0$. Since the function g is strictly increasing, we have that $\forall\, \alpha\ > \alpha_0, g(\alpha) > g(\alpha_0) = 0$. Hence $\zeta(\alpha) > 1\ \ \forall\, \alpha > \alpha_0$. □

C.5 Proof of Proposition 12

Consider a network with N_u users and N_c carriers. In the PRA scheme with ordered selection, each users selects one carrier with the maximum gain. Suppose that n users contend for a particular carrier. Also, let $\{h_k\}_{k=1}^n$ denote the i.i.d. instantaneous channel gain for the users. The probability that a particular packet is received successfully is given by

$$\Pr\left\{\frac{h_1}{N_0 + \sum_{k=2}^n h_k} > \lambda\right\}.$$

It is well-known that (see pp. 207 in [21]) that as $N_c \to \infty$, $h_1 - \ln N_c$ converges in distribution to $F(h) = e^{-e^{-h}}$. It can then be easily shown that

$$\lim_{N_c \to \infty} \Pr\left\{\frac{h_1}{\ln N_c} < h\right\} \quad = \quad \mathbb{1}_{\{h>1\}}. \tag{C.18}$$

Therefore, $\frac{h_1}{\ln N_c} \to 1$ in probability and hence

$$\frac{1}{n-1}\sum_{k=2}^{n} \frac{h_k}{\ln N_c} \to 1$$

in probability. It then follows that for any $N_0, \lambda > 0$, as $N_c \to \infty$, $\frac{h_1}{N_0+\sum_{k=2}^n h_k} \to \frac{1}{n-1}$ in probability and that

$$\Pr\left\{\frac{h_1}{N_0 + \sum_{k=2}^n h_k} > \lambda\right\} \to \mathbb{1}_{\{n-1 \le \lambda^{-1}\}}$$

which completes the proof. $\square$

REFERENCES

[1] B. C. Arnold, N. Balakrishnan, and H. N. Nagaraja, *A First Course in Order Statistics*. John Wiley, first ed., 1992.

[2] B. Hajek, A. Krishna, and R. O. LaMaire, "On the Capture Probability for a Large Number of Stations," *IEEE Trans. Commun.*, vol. 45, pp. 254–260, Feb. 1997.

[3] B. Hajek and T. van Loon, "Decentralized Bynamic Control of a Multiaccess Broadcast Channel," *IEEE Trans. Automatic Control*, vol. 27, pp. 559–569, June 1982.

[4] B. Tsybakov and W. Mikhailov, "Ergodicity of slotted Aloha systems," *Probl. Inform. Transm.*, vol. 15, no. 4, pp. 73–87, 1979.

[5] Bartle, R. G., *Elements of Real Analysis*. Wiley, second ed., 1975.

[6] C. Namislo, "Analysis of mobile radio slotted ALOHA networks," *IEEE JSAC*, vol. 2, July 1984.

[7] D. Bertsekas and R. Gallagher, *Data Networks*. Prentice Hall, second ed., 2000.

[8] D. Gesbert and M-S. Alouini, "How Much Feedback is Multi-User Diversity Really Worth?," in *Proc. ICC*, vol. 1, pp. 234 – 238, June 2004.

[9] D. J. Goodman and A. A. M. Saleh, "Local ALOHA radio communications with capture and packet buffers," in *Proc. GLOBECOM 1985*, Nov. 1985.

[10] FCC, "Spectrum policy task force report," *ET Docket No. 02-155*, Nov. 2002.

[11] G. D. Nyugen, A. Ephremides, andJ. W. Wieselthier, "Comments on "Capture and Retransmission Control in Mobile Radio"," *IEEE JSAC*, vol. 24, pp. 2340–2341, Dec. 2006.

[12] G. Ganesan, and Y. (G.) Li, "Channel aware aloha with imperfect csi," in *Proc. Globecom 2006, SFO*, Nov. 2006.

[13] G. Ganesan and Y. (G.) Li, "Stabilizing Multicarrier Aloha with Capture," *Submitted to IEEE Trans. Inform. Theory*, Aug. 2007.

[14] G. Ganesan, G. Song, and Y. (G.) Li, "Asymptotic throughput analysis of distributed multichannel random access schemes," in *Proc. IEEE ICC 2005, Seoul, South Korea*, pp. 3637 – 3641, May 2005.

[15] G. Ganesan, Y. (G.) Li, and A. Swami, "Channel Aware Aloha with Imperfect CSI," *Revised and resubmitted to IEEE Trans. Inform. Theory*, Sept. 2007.

[16] G. Ganesan, Y. (G.) Li, and F. W. Vook, "Stability region of multicarrier channel aware aloha," in *Proc. WCNC 2007, HK*, Mar. 2007.

[17] G. Ganesan, Y. (G.) Li, and F. W. Vook, "Stability Region of Multicarrier Channel Aware Aloha," *IEEE Trans. Inform. Theory*, Sept. 2007.

[18] G. Song and Y. (G.) Li, "Cross-layer optimization for OFDM wireless networks - Part I: Theoretical framework," *IEEE Trans. Wireless Commun.*, vol. 46, pp. 614–624, Mar. 2005.

[19] G. Song and Y. (G.) Li, "Cross-layer optimization for OFDM wireless networks - Part II: Algorithm development," *IEEE Trans. Wireless Commun.*, vol. 46, pp. 615–634, Mar. 2005.

[20] G. Thomas, "Capacity of wireless packet collision channel without feedback," *IEEE Trans. Inform. Theory.*, vol. 46, pp. 1141–1144, Jan. 2002.

[21] H. A. David, *Order Statistics*. New York: Wiley, first ed., 1970.

[22] I. M. I. Habbab, M. Kavehrad, and C-E. W. Sundberg, "Aloha with Capture over Slow and Fast Fading Radio Channels with Coding and Diversity," *IEEE JSAC*, vol. 7, pp. 79–88, Jan. 1989.

[23] J. C. Arnbak and W. van Blitterswijk, "Capacity of Slotted Aloha in Rayleigh-Fading Channels," *IEEE Trans. JSAC*, vol. 5, no. 2, pp. 261–269, 1987.

[24] J. M. Cioffi, "A Multicarrier Primer," Available at http://www.stanford.edu/group/cioffi/documents/multicarrier.pdf.

[25] K. L . Chung, *A First Course in Probability Theory*. Academic Press, second ed., 2000.

[26] L. Tong, V. Naware, and P. Venkitasubramaniam, "Signal processing in random access," *IEEE Signal Processing Mag.*, vol. 21, pp. 29–39, 2004.

[27] M. Medard, "The Effect upon Channel Capacity in Wireless Communications of Perfect and Imperfect Knowledge of the Channel," *IEEE Trans. Wireless Commun.*, vol. 46, pp. 933–946, May 2000.

[28] M. Ngo and V. Krishnamurthy, "Optimal Transmission Algorithms for Slotted ALOHA Sensor Networks with Multi-Packet Reception," in *Proc. ICASSP*, vol. 3, pp. 18–23, Mar. 2005.

[29] M. Zorzi and R. R. Rao, "Capture and Retransmission Control in Mobile Radio," *IEEE JSAC*, vol. 12, pp. 1289–1298, Oct. 1994.

[30] M. Zorzi and R. R. Rao, "Reply to "Comments on 'Capture and Retransmission Control in Mobile Radio'"," *IEEE JSAC*, vol. 24, pp. 2341–2342, Dec. 2006.

[31] Mitola, I. J., "Software radios: Survey, critical evaluation and future directions," *IEEE Aerosp. Electron. Syst. Mag.*, vol. 8, pp. 25–31, Apr. 1993.

[32] P. Venkitasubramaniam, S. Adireddy, and L. Tong, "Sensitivity and coding of oppurtunistic Aloha in Sensor networks with mobile access," *Journal of VSLI Signal Processing*, vol. 41, pp. 329–344, 2005.

[33] P. Viswanath, D. Tse, and R. Laroia, "Oppurtunistic beamforming using dumb antennas," *IEEE Trans. Inform. Theory.*, vol. 48, pp. 1277–1294, June 2002.

[34] R. L. Wheeden and A. Zygmund, *Measure and Integral.* Marcel Dekker Inc., first ed., 1977.

[35] R. M. Loynes, "The stability of a queue with nonindepedant inter-arrival and service times," *Proc. Cambridge Philos. Soc.*, vol. 58, pp. 497–520, 1962.

[36] R. Rao and A. Ephremides, "On the stability of interacting queues in a multi-access system," *IEEE Trans. Inform. Theory.*, vol. 34, pp. 918–930, Sept. 1979.

[37] S. Adireddy and L. Tong, "Optimal placement of training for frequency-selective block-fading channels," *IEEE Trans. Inform. Theory.*, vol. 48, pp. 2338–2353, Aug. 2002.

[38] S. Adireddy and L. Tong, "Exploiting decentralized channel state information for random access," *IEEE Trans. Inform. Theory.*, vol. 51, pp. 537–561, Feb. 2005.

[39] S. Ghez, S. Verdu, and S. Schwartz, "Stability Properties of Slotted Aloha with Multipacket Reception," *IEEE Trans. Automatic Control*, vol. 33, pp. 640–649, July 1988.

[40] S. Ghez, S. Verdu, and S. Schwartz, "Optimal Decentralized Control in the Random Access Multipacket Channel," *IEEE Trans. Automatic Control*, vol. 34, pp. 1153–1163, Nov. 1989.

[41] S. Sanayei, A. Nosratinia, and N. Aldhahir, "Oppurtunistic dynamic subchannel allocation in multiuser OFDM networks with limited feedback," in *Proc. IEEE Information Theory Workshop*, 2004.

[42] S. Sanayei and A. Nosratinia, "Exploiting Multiuser Diversity with Only 1-Bit Feedback," in *Proc. IEEE WCNC*, vol. 2, pp. 978 – 983, Mar. 2005.

[43] S. Shakkottai, T. S. Rappaport, and P. C. Carlson, "Cross-layer design for wirelss networks," *IEEE Commun. Magazine*, pp. 74–80, Oct. 2003.

[44] T. M. Cover and J. A. Thomas, *Elements of Information Theory.* Wiley-Interscience, first ed., 1991.

[45] V. F. Kolchin, B. A. Sevastyanov, and V. P. Chistyakov, *Random Allocations.* John-Wiley, first ed., 1977.

[46] V. Naware, G. Mergen, and L. Tong, "Stability and delay of finite-user slotted Aloha with multipacket reception," *IEEE Trans. Inform. Theory.*, vol. 51, pp. 2636–2656, July 2005.

[47] V. Srivastava, C. K. Ho, P. H. W. Fung, and S. Sun, "Robust mmse channel estimation in ofdm systems with practical timing synchronization," in *Proc. WCNC 2004*, vol. 2, pp. 711–716, Mar. 2004.

[48] V. Wong and C. Leung, "Capture Probability in a Mobile Packet Radio System," *IEEE Trans. Commun.*, vol. 40, pp. 1577–1580, Oct. 1992.

[49] W. Luo and A. Ephremides, “Stability of N interatcing queues in random-access systems,” *IEEE Trans. Inform. Theory.*, vol. 45, pp. 1579–1587, July 1999.

[50] W. Szpankowski, “Stability conditions for some multiqueue distributed systems: Buffered random access systems,” *Advances in Applied Probability*, vol. 26, pp. 498–515, 1994.

[51] X. Liu, E. K. P Chong, and N. B. Shroff, “Oppurtunistic transmission scheduling with resource sharing constraints in wireless networks,” *IEEE Journal Sec. Areas Commun.*, vol. 19, pp. 2053–2064, Oct. 2001.

[52] X. Qin and R. Berry, “Exploiting multiuser diversity for medium access control in wireless networks,” in *Proc. IEEE INFOCOM*, pp. 1084–1094, 2003.

[53] X. Qin and R. Berry, “Oppurtunistic splitting algorithms for wireless networks,” in *Proc. IEEE INFOCOM, Hong Kong, PR China*, 2004.

[54] Y.-H. Nam, C. Zhang, H. E-. Gamal, and A. Reid, “Opportunistic communications with distorted csit,” in *Proc. ISWCS 2006*, pp. 16–20, 2006.

[55] Y. Yu and G. B. Giannakis, “Oppurtunistic access in slotted aloha adapted to decentralized csi,” in *Proc. GLOBECOM 2004, Dallas, TX*, Nov. 2004.

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